MAC 3105-2A Print Name ______________________
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Midterm
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INSTRUCTIONS:

- Write answer in the space provided after the problems.
- Clearly show **ALL** work and circle/box answer.
- I_n is the $n \times n$ **Identity Matrix** of the required dimension.
- Keep Calm and Enjoy Linear

1. Definitions

Write down the definitions of the following terminologies.

(1) (5 pts) The vectors $\{u_1, u_2, u_3, u_4\}$ are *linear independent*.

(2) (5 pts) The Rank of a matrix A.

2. 'Trick or Treat'

Determine whether the statement is true or false. If it is true, say so; if it is false, explain why or give an example that disproves the statement.

(1) (5 pts) Let A, B, C be $n \times n$matrices, then

$$\det(A)\det(C) + \det(B)\det(C) = \det((A+B)C).$$

(2) (5 pts) Two different bases of a subspace have the same number of vectors.

(3) (5 pts) $\{(x, y, z) | x^2 + y^2 + z^2 = 0\}$ is **NOT** a linear subspace of $\mathbb{R}^3$.

(4) (5 pts) Any linear transform T will take 0 to 0.

(5) (5 pts) There exists **NO** injective linear transform from $\mathbb{R}^4$ to $\mathbb{R}^3$.

(6) (5 pts) If the Kernel of a linear transform $T\colon \mathbb{R}^4 \to \mathbb{R}^3$ is $\{0\}$, then the matrix that represents T has rank 3.

3. Hardcore Problems

3.1. **10 pts.** Find a basis for the Column space of A.

$$A = \begin{bmatrix} 1 & 4 & 8 & -3 & 7 \\ -1 & 2 & 7 & 3 & 4 \\ -2 & 2 & 9 & 5 & 5 \\ 3 & 6 & 9 & -5 & 2 \end{bmatrix}$$

3.2. **10 pts.** Find the LU decomposition for the following matrix.

$$A = \begin{bmatrix} 2 & 4 & -1 & 5 & -2 \\ -4 & -5 & 3 & -8 & 1 \\ 2 & -5 & -4 & 1 & 8 \\ -6 & 0 & 7 & -3 & 1 \end{bmatrix}.$$

3.3. **10 pts.** Use **Cramer's Rule** to solve the following matrix equation $AX = B$.

$$A = \begin{bmatrix} 1 & 4 & 5 \\ -2 & -7 & -8 \\ 4 & 9 & 6 \end{bmatrix}; B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}.$$

3.4. **10 pts.** Find the Inverse of the following matrix.

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 3 & 1 & 2 \\ 2 & 3 & 1 & 0 \\ 1 & 0 & 2 & 1 \end{bmatrix}$$

3.5. **10 pts.** Knowing that

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 12.$$

Find the determinant of the following matrix

$$A = \begin{bmatrix} 3d & 3e & 3f \\ 2a-g & 2b-h & 2c-i \\ 3d-g & 33-h & 3f-i \end{bmatrix}.$$

3.6. **15 pts.** Let $T\colon \mathbb{R}^5 \to \mathbb{R}^2$ be the linear transform that sends

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \mapsto \begin{bmatrix} 1 \\ 2 \\ 0 \\ 2 \end{bmatrix}; \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \mapsto \begin{bmatrix} 3 \\ 6 \\ 0 \\ 6 \end{bmatrix}; \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \mapsto \begin{bmatrix} -2 \\ -5 \\ 5 \\ 0 \end{bmatrix}; \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \mapsto \begin{bmatrix} 0 \\ -2 \\ 10 \\ 8 \end{bmatrix}; \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \mapsto \begin{bmatrix} 2 \\ 4 \\ 0 \\ 4 \end{bmatrix}.$$

Find a basis for the Kernel space of T.