

MAC 3105-2A  
Quiz 7  
07/06/2017

Print Name \_\_\_\_\_

Signature \_\_\_\_\_

INSTRUCTIONS:

- Write answer in the space provided after the problems.
- Clearly show ALL work and circle/box answer.
- $I$  is the **Identity Matrix** of the required dimension.
- Keep Calm and Enjoy Linear

UNLESS SPECIAL INSTRUCTED, WE ONLY CONSIDER **REAL** NUMBERS.

1. **True of False.** Let  $A$  be a  $4 \times 4$  **Real** matrix.

(1) The dot product of two vectors is always non-negative.

(2) The length of a vector is always positive.

(3) If  $\{u_1, u_2, u_3\}$  is an orthogonal set, then the matrix whose column vectors are  $u_1, u_2, u_3$  has rank 3.

(4) Let  $H \subset V$  be a linear subspace, then  $\dim H + \dim H^\perp = \dim V$ .

**2.** Let  $u = [1, 2, 3, 4]^T$ ,  $u_1 = [1, 1, -1, -1]^T$  and  $u_2 = [0, 1, -1, 2]^T$ . Let  $H$  be the plane generated by  $u_1, u_2$ . Find  $u_H$  and  $u_{H^\perp}$ .

3. Let  $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$  be a linear transform that is represented by

$$A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & -1 \\ 1 & 0 & -2 \end{bmatrix}$$

- (1) Show that the column vectors of  $A$  form an orthogonal set.
- (2) Show that the row vectors of  $A$  does NOT form an orthogonal set.
- (3) Find  $A^T A$ . Notice that it's NOT equal to  $AA^T$ .

4. Find the distance from the point  $(1, 1, 1, 1)$  to the plane  $x + y + z + w = 0$  in  $\mathbb{R}^4$ .