MAC 3105-2A
Quiz 7
07/06/2017

Print Name ____________________

Signature ____________________

INSTRUCTIONS:

- Write answer in the space provided after the problems.
- Clearly show ALL work and circle/box answer.
- I is the **Identity Matrix** of the required dimension.
- Keep Calm and Enjoy Linear

UNLESS SPECIAL INSTRUCTED, WE ONLY CONSIDER **REAL** NUMBERS.

1. True of False. Let A be a 4×4 **Real** matrix.

(1) If the eigenvalues of A are $1, 4, 0, 1$, then A is invertible.

(2) If the eigenvalues of A are $1, 2, 3, 4$, then A is diagonalizable.

(3) If u is a complex eigenvector of A, then $\bar{u}$ is also a complex eigenvector of A.

(4) Any invertible matrix is diagonalizable.

2. Let A be a 3×3 matrix as follows.

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 5 & 1 \end{bmatrix}$$

Find all the real and complex eigenvalues of A. Find a basis for each eigenvalue.

3. Let $T\colon \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transform that is represented by

$$A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & -1 \\ 1 & 0 & -2 \end{bmatrix}$$

Find the matrix representation of T under the basis

$$\{ \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} ; \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} ; \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix} .\}$$

4. Find a diagonalization of the following matrix.

$$A = \begin{bmatrix} 1 & 1 & 2 \\ -1 & 1 & -2 \\ 1 & 0 & 2 \end{bmatrix}$$

1. Solutions

1. F. The eigenvectors of 0 is the Null space of A / T / T / F. Show them the Jordan block. It is invertible, but not diagonalizable.

2. Trivial

3. It should be the inverse of A, since the column vectors are exactly the basis vectors.

4. Find the eigenvalues, this will give you the diagonal matrix D. Then to find the transform matrix P, just find bases for each eigenvalue. P is composed by the the basis vectors.