

MAC 3105-2A  
Quiz 7  
07/06/2017

Print Name \_\_\_\_\_

Signature \_\_\_\_\_

UNLESS SPECIAL INSTRUCTED, WE ONLY CONSIDER **REAL** NUMBERS.

1. Let  $A$  be a  $5 \times 5$  matrix as follows.

$$\begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & -2 & 1 \end{bmatrix}$$

- (1) Find all the eigenvalues of  $A$ .
- (2) For each eigenvalue, find a basis for its eigen-space.
- (3) Is  $A$  diagonalizable? If not, Explain why. If it is, find the matrix  $P$  and  $D$  such that  $A = PDP^{-1}$ . Here  $D$  is diagonal, and  $P$  is an invertible matrix.

2. Let  $A$  be a  $3 \times 3$  matrix as follows.

$$\begin{bmatrix} 2 & 3/2 & 0 \\ -2 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- (1) Find the diagonalization of  $A$ , i.e., find an invertible matrix  $P$  and a diagonal matrix  $D$  such that  $A = PDP^{-1}$ .
- (2) Find  $A^{2017}$ , i.e.,  $A \times A \times \cdots \times A$  multiplied 2017 times.

3.  $A$  is a  $5 \times 5$  matrix that has eigenvalue 2 of multiplicity 5, and is diagonalizable. List all the possible  $A$ 's.