MAC 3105-2A Print Name ______________________
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Quiz 4 & 5
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INSTRUCTIONS:

- Write answer in the space provided after the problems.
- Clearly show ALL work and circle/box answer.
- I is the **Identity Matrix** of the required dimension.
- Keep Calm and Enjoy Linear

1. Definitions

Write down the definition of the following terminologies. Let $T\colon \mathbb{R}^n \to \mathbb{R}^m$ be a linear transform.

(1) (5 pts) The Basis of a linear subspace H.

(2) (5 pts) Column space of a matrix A.

2. 'Trick or Treat'

Determine whether the statement is true or false. If it is true, say so; if it is false, explain why or give an example that disproves the statement.

(1) (5 pts) Let A, B, C be $n \times n$matrices, then $(A + B)C = AC + BC$.

(2) (5 pts) Let $T\colon \mathbb{R}^3 \to \mathbb{R}^4$ be a linear transform. If $T(u_1)$, $T(u_2)$ and $T(u_3)$ are linear independent, then u_1, u_2, u_3 are linear independent.

(3) (5 pts) The basis of a linear subspace is unique.

(4) (5 pts) $\{(x, y)|x - y = 0\}$ is **NOT** a linear subspace of $\mathbb{R}^2$.

(5) (5 pts) If the Kernel of a linear transform $T\colon \mathbb{R}^3 \to \mathbb{R}^3$ is $\{0\}$, then the matrix represents T has rank 3.

(6) (5 pts) Only a square matrix has a LU decomposition.

3. Hardcore Problems

3.1. **10 pts.** Solve the following linear system, and parametrize the solution space.

$$\begin{cases} x + y + 5z = 1 \\ 2x - 2y - 2z = -2 \\ -x + 3y + 5z = 3 \end{cases}$$

3.2. **10 pts.** Is the vector $u = [-4, 10, -7, 5]^T$ in the column space of A ? **Explain** why or why not.

$$A = \begin{bmatrix} 1 & 4 & 5 \\ -2 & -7 & -8 \\ 4 & 9 & 6 \\ 3 & 7 & 5 \end{bmatrix}$$

3.3. **10 pts.** Find the LU decomposition of the following matrix A if it exists.

$$A = \begin{bmatrix} 1 & 4 & 2 & 3 \\ 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

3.4. **10 pts.** Let A be a partitioned matrix

$$A = \begin{bmatrix} A_{1,1} & A_{1,2} \\ A_{2,1} & A_{2,2} \end{bmatrix}.$$

Here

$$A_{1,1} = \begin{bmatrix} 1 & 4 \\ 1 & 2 \end{bmatrix}; A_{1,2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}; A_{2,2} = \begin{bmatrix} 0 & 0 \end{bmatrix}; A_{2,2} = \begin{bmatrix} 1 & 0 \end{bmatrix};$$

Find a partition for

$$A_{1,2} = \begin{bmatrix} 1 & 3 \\ -1 & 1 \\ 1 & 0 \\ 2 & 1 \end{bmatrix},$$

and find AB using the matrix multiplication for partitioned matrices.

3.5. **15 pts.** Let $T\colon \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transform that sends $[2,2,2]^T$ to $[2,0,0]^T$, $[-1,1,2]^T$ to $[0,1,0]^T$, and $[1,1,4]^T$ to $[0,0,1]^T$. Find the matrix A such that A represents T.

3.6. **10 pts.** Let $X = [x, y]^T$. Let

$$A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix},$$

Find a basis for the solution space to the following equation.

$$X^T A X = 0$$