# BASIC DEFINITIONS

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## 1. Chapter I

**Linear Independent.** A set of vectors $\{u_1, u_2, \cdots u_k\}$ are *linear independent* if the only solution to $a_1u_1 + a_2u_2 + \cdots + a_ku_k = 0$ is $a_1 = a_2 = \cdots = a_k = 0$.

**Span Space.** The *Span Space* of vectors $\{u_1, u_2, \cdots u_k\}$ is the space that contains all the linear combinations of them.

**Linear Transform.** A *Linear Transform* is a function $T : \mathbb{R}^n \to \mathbb{R}^m$ such that

(1) For any vectors $u$ and $v$, $T(u) + T(v) = T(u + v)$.
(2) For any scalar $c$, $T(cu) = cT(u)$

**Injective and Surjective.** Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transform.

(1) A linear transform $T$ is *injective* if whenever $u \neq v$, $T(u) \neq T(v)$. Or equivalently, $T(u) = 0$ if and only if $u = 0$; i.e., the kernel space of $T$ is $\{0\}$.
(2) A linear transform $T$ is *surjective* if for any vector $B$ in $\mathbb{R}^m$, there exists some $u$ in $\mathbb{R}^n$ such that $T(u) = B$. Or equivalently, the image space of $T$ is $\mathbb{R}^m$.

**Kernel Space and Image Space.** Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transform.

(1) The *Kernel Space* of $T$ is the set of vectors $u$ in $\mathbb{R}^n$ such that $T(u) = 0$.
(2) The *Image Space* of $T$ is the set of vectors $B$ in $\mathbb{R}^m$ such that $B = T(u)$ for some $u$ in $\mathbb{R}^n$.

## 2. Chapter II

**Column Space.** The *Column Space* of a matrix $A$ is the space spanned by the column vectors of $A$.

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**(Linear) Subspace.** A *(Linear) Subspace* of $\mathbb{R}^n$ is a subset $H$ of $\mathbb{R}^n$ such that

(1) If $u$ and $v$ are in $H$, then $u + v$ is in $H$.
(2) If $u$ is in $H$, then $cu$ is in $H$ for any scalar $c$.

**Basis of a Subspace.** The *Basis* of a linear subspace $H$ is a set of vectors $\mathcal{B}$ such that

(1) The vectors in $\mathcal{B}$ span $H$.
(2) The vectors in $\mathcal{B}$ are linear independent.

**Dimension of a Linear subspace.** The *dimension* of a linear subspace is the number of vectors in any of its basis.

**Rank of a Matrix.** The *Rank* of a matrix is the dimension of the column space of $A$. Or equivalently, the number of linear independent vectors among the column vectors.

## 3. Final Exam

THe definitions of the final exam will come from the following.

(1) $\lambda$ is an eigenvalue of $A$.
(2) $u$ is an eigenvector of $A$.
(3) Characteristic polynomial of a matrix $A$.
(4) $A$ is diagonalizable.
(5) The dote product of $u$ and $v$. The length of $u$. The distance between $u$ and $v$.
(6) The orthogonal projection of $u$ on a subspace $H$.
(7) A set of vectors is orthogonal/orthonormal.
(8) Unit vector
(9) A matrix is symmetric.
(10) A quadratic form/symmetric matrix is positive definite/negative definite/indefinite.