Introduction to Calculus III

Vector Calculus

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January 9, 2017

Outline

Syllabus and Special Instructions

From One Variable to Several Variables

- One Variable Calculus
- Several Variables

What are we going to learn this Semester

- Vectors
- Vector Calculus

About the Course

We will learn how to generate one variable function theory into multipli-variables using vector calculus.
Here is the course website:
http://www.math.fsu.edu/ xzhang/en/mac2313sp17/

About Me

This guy is lazy.

Eligibility

You must have the course prerequisites listed below and must never have completed with a grade of C- or better a course for which MAC 2313 is a (stated or implied) prerequisite. Students with more than eight hours of prior credit in college calculus are required to reduce the credit for MAC 2313 accordingly. It is the student's responsibility to check and prove eligibility.

PREREQUISITES

You must have passed MAC 2312 (Calculus II) with a grade of C- or better or have satisfactorily completed at least eight hours of calculus courses equivalent to MAC 2311 and MAC 2312.

Homework and Test

1. Attendance is NOT mandatory.
2. There will be a homework every week. The homework will be named like 'Homework Week 01', and will be due by 11:59 pm every Sunday without special instructions.
3. There will be four unit tests.
   Test#1: Thursday, February 2.
   Test#2: Thursday, February 23.
   Test#3: Thursday, March 23.
   Test#4: Thursday, April 20.
4. There will be a Final Exam.
   Final Exam: Wednesday, May 3rd, 05:30 - 07:30 pm
5. **All tests and final exam will be given in LOV 102**

Grading

The formula of the numerical grade is

$$(5T + H + 4F)/10$$

Here T = unit test average, H =homework and quiz average, and F = final exam. Letter grades will be determined from numerical grade according to the syllabus.

Some Suggestions

1. There is ONE rule: DO NOT INTERRUPT OTHERS.
2. If you have any questions with math, ASK. There are NO dumb questions with math.
3. Make sure you are **understanding** the materials.
4. Do the homework by yourself first, before you discuss with others.
5. Keep a good record of the lecture notes.

Welcome

Welcome to the first REAL math course !

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Here the journey starts !

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Function $y = f(x)$

Let $y = f(x)$ be a function. The graph of $y = f(x)$ is a plane curve, i.e., a one-dimensional object. We have talked about the following math concepts

1. Limit
2. Continuity
3. Differentiation
4. Riemann Sum and Integration

Application of Differentiation

We also talked about the applications of differentiation.

1. 1st derivative $f'(x) \Longleftrightarrow$ tangent direction $\Longleftrightarrow$ rate of change
2. sign of $f'(x) \Longleftrightarrow$ increasing / decreasing of $f(x)$.
3. sign of $f''(x) \Longleftrightarrow$ concavity of $f(x)$.
4. Maximal / Minimum problem.

Application of Integration

And we talked about why do we need integration

1. Integration gives the area bounded by curves.
2. Integration gives the volume of a cylindrical object.
3. arc length of a parametrized curve .
4. area of a parametrized surface.
5. Any physical quantity computed by adding up small pieces uses integration.

Application of Integration

And we talked about how to integrate:

Calculus II

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Why several variables I

Example

Find the distance between $(0,0,0)$ to the plane $x+y+z=1$.

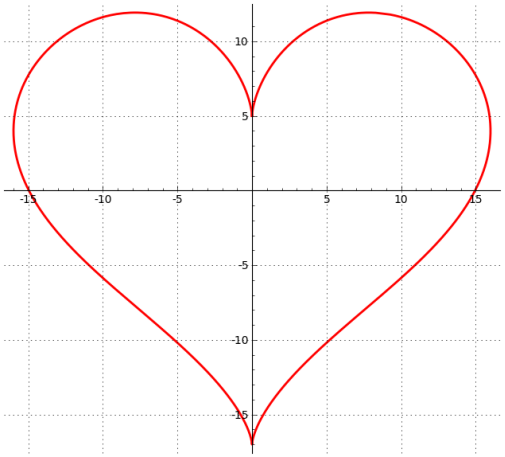
1. The distance between a point (x,y,z) to $(0,0,0)$ is $d(x,y,z)=\sqrt{x^2+y^2+z^2}$.
2. The distance between $(0,0,0)$ to the plane is the minimum value of this distance function with restriction $x+y+z=1$.
3. Lagrangian Method provides a way to solve it.

What do we need

To complete the above process, as the 'similar' analog to one variable case, we will use the 1st partial derivatives (Critical Points) and 2nd partial derivatives (2nd Derivative Test), together with the **BOUNDARY** condition.

Why Several Variables II

Example



Find the energy change of an electron flying through the curve in a electronic force field.

How to solve

1. Find the CURL of the electronic vector field: $curl\vec{F} = 0$.
2. Apply the 'Poled' Green's Theorem, the line integral doesn't depend on the path.
3. The heart curve can be replaced by any smooth closed curve around the origin.
4. Replace it by the unit circle.
5. Do the line integration.

What do we need

To complete the above process, we need to be familiar with double integration, line integration and surface integration, and a decent understanding of Green's Theorem.

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Why Vectors:

1. $\vec{x} =< a, b, c >$ describes the point (a, b, c) in n-space, $\mathbb{R}^n$.
2. Vector form gives better expression of line / plane equations.
3. Vectors can help solve analytic geometry problems: much easier than analytic ways.
4. Most physics terminologies are vectors, instead of numbers: they have directions.

Space Curves

1. Tangent vectors and arc length
2. Frenet Frame: local frame of a curve.
3. Curvature: how far is the curve away from being a straight line.
4. Torsion: how far is the curve away from being a plane curve.

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Several Variable Functions

1. Domain
2. Limit: very different from what you knew.
3. continuity
4. Differentiability

Differentiation

1. Partial derivatives: Gradient vector field ∇f and Hessen matrix H_f.
2. Derivative rules: Chain rule and Implicit differentiation.
3. Maximum / Minimum problem: free version.
4. Maximum / Minimum problem: restricted version.

Integration with Functions

1. Double Integral.
2. Triple Integral.
3. Changing Variables.
4. Type I line / surface integral
5. Geometry meaning: area and volume.

Integral with Vector Fields

1. Type II line / surface integral.
2. Conservative vector field and the Fundamental Theorem of Calculus.
3. Curl and divergence
4. **Green's Theorem, Stoke's Theorem, and the Divergence Theorem.**

Thank You

Thank You and Good Night