

* How to Compute the determinant of a 3×3 matrix

— The determinant of a 2×2 matrix is given by:

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

Exp: $\begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} = 2 \times 1 - 2 \times 1 = 0$

$$\begin{vmatrix} 1 & 4 \\ -1 & 1 \end{vmatrix} = 1 \times 1 - 4 \times (-1) = 1 + 4 = 5$$

— The determinant of a 3×3 matrix

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Step 1: Pick any row or any column

for example: $(a_2 \ b_2 \ c_2)$ ← the 2nd row.

Step 2: From left to right, assign a sign for each number in the row by the following rule: $(-1)^{r+c}$ ← which column the number lives

← which row the number lives

for example: $a_2 \rightarrow (-1)^{2+1}$ $b_2 \rightarrow (-1)^{2+2}$ $c_2 \rightarrow (-1)^{2+3}$

Step 3: For each number in the row, find the 2×2 matrix after kicking out the row and the column the number lives.

Exp: For a_2 , the 2×2 matrix is $\begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix}$

for b_3 , the 2×2 matrix is $\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$

Step 3: For each number in the row,

multiply the sign with the determinant of the 2×2 matrix
~~the~~ number itself

Add up all the multiplications in the row.

Exp: $\begin{vmatrix} 1 & 2 & 3 \\ -1 & 1 & 0 \\ 4 & 2 & 1 \end{vmatrix}$ Pick this column

$$= (-1)^{1+2} \times \underline{2} \times \begin{vmatrix} -1 & 0 \\ 4 & 1 \end{vmatrix} + (-1)^{2+2} \times \underline{1} \times \begin{vmatrix} 1 & 3 \\ 4 & 1 \end{vmatrix} \\ + (-1)^{3+2} \times \underline{2} \times \begin{vmatrix} 1 & 3 \\ -1 & 0 \end{vmatrix}$$
$$= (-2) \times (-1) + 1 \times 1 \times (-11) + (-1) \times 2 \times 3$$
$$= 2 - 11 - 6 = -15.$$