

# XP's Notes

## < Solving linear system of equations >

\* Case One: Number of equations < number of variables  
 Infinite many solutions

\* Case Two: Number of equations = number of variables.

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = a_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = a_2 \\ \vdots \\ a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = a_n \end{cases} \iff \begin{matrix} \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix} & \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} & = & \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \\ \text{A} & \text{X} & = & \text{B} \end{matrix}$$

denoted by  $AX = B$

$\det A \neq 0 \Rightarrow$  unique solution

$\det A = 0 \Rightarrow$  No solution or infinite solution.

In the second case, set up the augmented matrix

$$\bar{A} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} & | & a_1 \\ a_{21} & a_{22} & \dots & a_{2n} & | & a_2 \\ \vdots & \vdots & \ddots & \vdots & | & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} & | & a_n \end{bmatrix} \quad \text{then} \quad \begin{cases} \text{if there is a row like } (0, 0, \dots, 0, | a) \\ \quad a \neq 0 \\ \text{otherwise} \end{cases} \begin{matrix} \Rightarrow \text{No solution} \\ \\ \Rightarrow \text{infinite solutions.} \end{matrix}$$

reduces  $\bar{A}$  to echelon form

\* Case Three: Number of equations > number of variables. ( $m > n$ )

Form the augmented matrix

$$\bar{A} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} & | & a_1 \\ a_{21} & a_{22} & \dots & a_{2n} & | & a_2 \\ \vdots & \vdots & \ddots & \vdots & | & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} & | & a_m \end{bmatrix} \rightarrow \text{reduce to echelon form}$$

$$\begin{cases} \text{if there is a row like } (0, 0, \dots, 0, | a) \\ \quad a \neq 0 \\ \text{otherwise} \end{cases} \begin{matrix} \Rightarrow \text{No solution} \\ \\ \left\{ \begin{array}{l} rk \bar{A} = n \Rightarrow \text{unique solution} \\ rk \bar{A} < n \Rightarrow \text{Infinite solutions} \end{array} \right. \end{matrix}$$