MAC 2311-0003
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Test 2
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Print Name XP's Answers

Signature ____________________

INSTRUCTIONS:

- Clearly show all work where instructed.
- Write answer in space provided or circle/box answer.

KEEP CALM AND DO SOME CALCULUS !

1.(15pts). Find the first derivatives of the following functions:

(1) (5pts) $f(x) = e^{x^2+1}$

$$f'(x) = \underbrace{e^{x^2+1}}_{3pts} \cdot \underbrace{2x}_{2pts}$$

(2) (4pts) $f(x) = \arcsin(x)$

$$f'(x) = \frac{1}{\sqrt{1-x^2}}$$

(3) (6pts) $f(x) = x\ln(2x)$

$$f'(x) = \underbrace{\ln(2x)}_{3pts} + \underbrace{1}_{3pts}$$

2.(15pts). Given the table of values, find the quantities blow:

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
2	3	-3	5	-2
3	2	-2	4	3
5	2	6	3	-3

(1) (5pts) $(f/g)'(2) =$ $\frac{-9}{25}$

(2) (5pts) $(f \circ g)'(2) =$ -12

(3) (5pts) Let $h(x) = x^2 f(x)$, then $h'(3) =$ -6

3.(8pts). Find the first derivative of the function $f(x) = \arctan(\sqrt{x^2 - 4})$. **Simplify** your answer as much as possible.

$$f'(x) = \frac{1}{1+(\sqrt{x^2-4})^2} \cdot (\sqrt{x^2-4})' \quad \Big\} \text{ 3 pts}$$

$$= \frac{1}{1+x^2-4} \cdot \frac{1}{2} \cdot (x^2-4)^{\frac{-1}{2}} \cdot (x^2-4)' \quad \Big\} \text{ 3 pts}$$

$$= \frac{1}{x^2-3} \cdot \frac{1}{2} \cdot (x^2-4)^{\frac{-1}{2}} \cdot 2x \quad \Big\} \text{ 1 pt}$$

$$= \frac{x}{x^2-3} \cdot \frac{1}{\sqrt{x^2-4}} = \frac{x}{(x^2-3)(\sqrt{x^2-4})} \quad \Big\} \text{ 1 pt}$$

4.(8pts). Use logarithmic differentiation to find the first derivative of the following function. Show work and circle your answer.

$$f(x) = (x^2 + 1)^{\sin(x)}$$

Use logarithmic differentiation.

$$\ln f(x) = \ln (x^2+1)^{\sin x} = \sin x \cdot \ln(x^2+1)$$

$$\frac{d}{dx} \ln (f(x)) = \frac{d}{dx}(\sin x \cdot \ln (x^2+1)) \quad \Big\} \text{ 3 pts}$$

$$\frac{f'(x)}{f(x)} = \cos x \cdot \ln (x^2+1) + \sin x \cdot \frac{1}{x^2+1} \cdot (x^2+1)'$$

$$\frac{f'(x)}{f(x)} = \cos x \cdot \ln(x^2+1) + \frac{2x \cdot \sin x}{x^2+1} \quad \Big\} \text{ 4 pts}$$

$$f'(x) = (x^2+1)^{\sin x} \cdot \left(\cos x \cdot \ln (x^2+1) + \frac{2x \cdot \sin x}{x^2+1}\right) \quad \Big\} \text{ 1 pt}$$

5.(8pts). Use implicit differentiation to find $\frac{dy}{dx}$ if $xy + 3e^{x+y} = 3$. Clearly show the work and circle your answers. **Simplify** your answer as much as possible.

$$\frac{d}{dx}(xy + 3e^{x+y}) = \frac{d}{dx}(3)$$

$$y + x\cdot y' + 3e^{x+y}\cdot(1+y') = 0$$

5pts

$$(y+3e^{x+y}) + (x+3e^{x+y})y' = 0$$

1pt

$$y' = \frac{-(y+3e^{x+y})}{x+3e^{x+y}}$$

2 pts

6.(16pts). Clearly show the work and circle your answers. **Simplify** your answers as much as possible.

A curve is given by the equation $x^4 + y^4 = 16$.

(1) (6pts) Find $\frac{dy}{dx}$, the first derivative of y with respect to x.

$$\frac{d}{dx}(x^4+y^4) = \frac{d}{dx}(16)$$

$$4x^3 + 4y^3y' = 0$$

4pts

$$4y^3y' = -4x^3$$

$$y' = \frac{-x^3}{y^3}$$

2pts

(2) (4pts) Find the equation of the line tangent to the curve at $x = 2$.

when $x=2$, $2^4 + y^4 = 16 \Rightarrow y = 0$

so the pt is $(2, 0)$

2pts

slope $= y'|_{(2,0)} = \frac{-2^3}{0^3} = $ D.N.E $(=\infty)$

So this is a vertical line.

$$x = 2.$$

2pts

(3) (6pts) Find $\frac{d^2y}{dx^2}$, the second derivative of y with respect to x.

$$y'' = (y')' = \frac{(-x^3)'y^3 - (-x^3)\cdot(y^3)'}{y^6} = \frac{-3x^2y^3 + 3x^3y^2\cdot y'}{y^6}$$

3 pts

$$= \frac{-3x^2y^3 + 3x^3y^2\cdot\frac{-x^3}{y^3}}{y^6} = \frac{-3x^2y^6 + (-3)x^6y^2}{y^9} = \frac{-3x^2y^4 - 3x^6}{y^7}$$

[redacted] 2pts [redacted] 1pt

7.(18pts). Show the work and circle your answers.

A particle moves on a straight line so that its displacement s at time t is given by

$$s(t) = t^3 + 3t^2 - 24t.$$

Here time is considered to be non-negative.

(1) (3pts) Find the velocity function.

$$\underbrace{v(t) = s'(t)}_{1\text{ pt}} = \underbrace{3t^2 + 6t - 24}_{2\text{pts}}$$

(2) (2pts) Find the acceleration function.

$$\underbrace{a(t) = v'(t)}_{1\text{ pt}} = 6t + 6$$

(3) (3pts) When is the particle at rest?

$$\left.\begin{aligned} &\text{at rest} \Leftrightarrow v(t) = 0 \\ &3t^2 + 6t - 24 = 0 \\ &3(t^2 + 2t - 8) = 0 \end{aligned}\right\} \text{2pts}$$

$$\left.\begin{aligned} &3(t-2)(t+4) = 0 \\ &t = 2 \text{ or } t = -4 \\ &\text{But } t \geq 0, \text{ so } t = 2 \end{aligned}\right\} \text{1 pt}$$

(4) (5pts) When is the particle speeding up?

sign graph for $v(t)$ & $a(t)$

	0 to 2	2	$t > 2$
$v(t)$	$-$		$+$
$a(t)$	$+$		$+$

4 pts

so when $\underbrace{t > 2}_{1\text{ pt}}$, it's speeding up

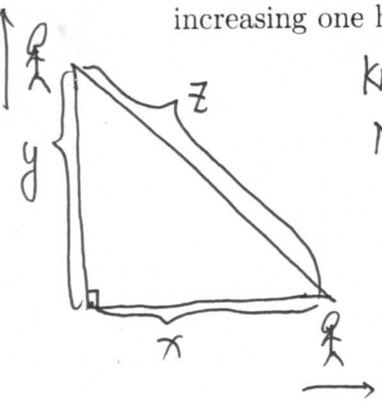
(5) (5pts) Find the total distance traveled by the particle in the first 3 seconds. Show work or explain.

$$\text{Distance} = \left.|s(2) - s(0)| + |s(3) - s(2)|\right\} \text{3 pts}$$

$$= |(2^3 + 3\cdot 2^2 - 24\cdot 2) - 0| + |(3^3 + 3\cdot 3^2 - 24\cdot 3) - (2^3 + 3\cdot 2^2 - 24\cdot 2)|$$

$$= |-28| + |-18 - (-28)| = 28 + 10 = 38$$

8.(12pts). Two men start walking from the same point. One walks north at $5mi/hr$ and the other walks east at $3mi/hr$. At what rate is the distance between the men increasing one hour later?



Known: $\frac{dx}{dt}=3$. $\frac{dy}{dt}=5$.
Need: $\left.\frac{dz}{dt}\right|_{t=1}=?$ — 4 pts

relation: $x^2+y^2=z^2$ — 2 pts

implicit differentiation:
$$2x\cdot\frac{dx}{dt}+2y\cdot\frac{dy}{dt}=2z\cdot\frac{dz}{dt}$$
solve for $\frac{dz}{dt}$: $\frac{dz}{dt}=\frac{x\cdot\frac{dx}{dt}+y\cdot\frac{dy}{dt}}{z}$ — 3 pts

When $t=1$. $x=3$, $y=5$. $z=\sqrt{3^2+5^2}=\sqrt{34}$ — 2 pts

so $\left.\frac{dz}{dt}\right|_{t=1}=\frac{3\cdot3+5\cdot5}{\sqrt{34}}=\sqrt{34}$ — 1 pt

9.(5pts). Newton's Law of Gravitation says that the magnitude F of the force exerted by a body of mass m on a body of mass M is
$$F=\frac{GmM}{r^2}.$$
Where G is the gravitational constant and r is the distance between the bodies. Find the rate of change of the magnitude with respect to the distance assuming the masses are constants.

$F=GmM\cdot r^{-2}$.

$\frac{dF}{dr}=GmM\,(r^{-2})'$ — 2 pts

$=GmM(-2)\cdot r^{-3}$

$\frac{dF}{dr}=\frac{-2GmM}{r^3}$ — 3 pts