1. 3.9 Related Rates

Recall: If one quantity, say y, varies according to another quantity, say t, then the instantaneous rate of change of the quantity y with respect to the quantity t is $\frac{dy}{dt}$.

Convention: The rate of change of a quantity is understood to be with respect to time unless otherwise specified.

General Steps:

Step 1. Read problem quickly to get an idea about whether there are any picture you can draw or formulas that you will need to use. Draw those pictures and write down the formulas.
Step 2. Reread the problem carefully and express all information from the problem mathematically. Use variables to represent any quantity that changes. Numbers may be used for quantities that remain constant. Be sure to note which quantities change and which do not.
Step 3. Find an equation that relates the quantities discussed in the problem.
Step 4. Use implicit differentiation to get an equation that relates rates of changes from the equation in 3.
Step 5. Plug in the values that were variable.
Step 6. Re-read the problem and answer the question.

2. Examples

Example 2.1. *If a spherical ballon is being blown up such that the diameter is increasing at a rate of* 2 *cm/sec, find the rate the surface area is increasing when the surface area is* 36π *cm*^{2}.

Example 2.2. *A cone is growing at a rate of 5 in*^{3}*/min and the radius is growing at a rate of 4 in/min. Find how fast the height is changing if the radius and height are both 10 in. Is the height increasing or decreasing?*

3. Examples - Includes Finding a Formula

Example 3.1. *As a man walks away from a 12 ft lamppost, the tip of his shadow moves twice as fast as he does. What is the man's height?*

Example 3.2. *A baseball diamond is a square with side 90 ft.*

(1) *A batter hits the ball and runs towards first base with a speed of 22 ft/s. How fast is his distance from second base changing when he is halfway to first base?*

(2) *A player is running from 2nd base to 3rd base and the catcher is standing on home plate watching that player. When the player running is halfway between the bases, the angle at the catcher from the runner to third base is decreasing at $\pi/6$ radians per second. How fast is the runner running?*

Example 3.3. *A right circular cone has a height that is decreasing while and the radius at the base of the cone is increasing. If the radius is increasing at 2 cm/sec when the height is 9 cm and the radius is 6 cm, how fast must the height be decreasing if the volume is to remain constant?*

Example 3.4. *A paper cup in the shape of an inverted right circular cone is being filled with water. The cup has height 10 cm and the radius at the top of the cup is 5 cm. If the cup is being filled at a rate of 1 cm*^{3}*/sec, how fast is the water level changing when it is 4 cm?*

Example 3.5. *A spotlight is located 3 m away from the nearest point P on the wall of a high rise and its light makes four revolutions per minute. How fast is the beam of light moving along the wall when it is 1 m from P?*

Example 3.6. *The minute hand on a clock is 12 cm long and the hour hand is 8 cm long. How fast is the distance between the tips of the hands changing at 2 o'clock?*