1. Section 3.7 Recall

- The instantaneous **velocity** of a particle with position at time t given by $s(t)$ at time t is...

- The instantaneous **acceleration** of a particle with position at time t given by $s(t)$ at time t is...

- The **jerk** of a particle with position at time t given by $s(t)$ at time t is...

- The **Average Rate of Change** of $y = f(x)$ from $(a, f(a))$ to $(x, f(x))$ is ...

- The **Instantaneous Rate of Change with respect to** x of $y = f(x)$ at $(a, f(a))$ is ...

2. Some new applications of derivatives

- The **linear density**, ρ, is the instantaneous rate of change of the mass function with respect to the position. i.e. $\rho = m'(x)$ if $m(x)$ represents the mass at position x.
- The **current**, I, is the instantaneous rate of change of the charge function with respect to time. i.e. $I = Q'(t)$ if $Q(t)$ represents the charge at time t.
- The **instantaneous rate of reaction**, of a product C is the instantaneous rate of change of the concentration function with respect to time. i.e. instantaneous rate of reaction $= [C]'(t)$ if $[C](t)$ represents the concentration at time t.
- The **compressibillity**, β, of a substance is the instantaneous rate of change of the volume function with respect to pressure multiplied by the negative reciprocal of the volume. i.e. $\beta = -\frac{1}{V(P)}V'(P)$ if $V(P)$ represents the concentration at pressure P.

3. Examples

Example 3.1. *A particle moves according to a law of motion* $s = f(t) = te^{-t/2}$, $t \geq 0$, *where* t *is measured in seconds and* s *in feet.*

1. *Find the velocity at time* t.
2. *What is the velocity after 1 s?*
3. *When is the particle at rest?*
4. *When is the particle moving in the positive direction? Give solution in interval notation.*
5. *Find the total distance traveled during the first 6 s.*
6. *Find the acceleration at time* t *and after 1 s.*
7. *When is the particle's acceleration positive?*
8. *When is the acceleration negative?*
9. *When is the particle speeding up? (when is the velocity and the acceleration in the same direction)*
10. *When is the particle slowing down? (when is the velocity and the acceleration in the opposite direction)*

4. Beginning Related Rates or Advanced Implicit Differentiation

Example 4.1. *The volume of a right circular cone is* $V = \frac{1}{3}\pi r^2 h$. *Find the rate of change of the volume with respect to the height if the radius is constant.*

Example 4.2. *The volume of a right circular cone is* $V = \frac{1}{3}\pi r^2 h$. *Find the rate of change of the volume with respect to the radius if the height is constant.*

Example 4.3. *The volume of a right circular cone is* $V = \frac{1}{3}\pi r^2 h$. *Find the rate of change of the volume in terms of* r, h, $\frac{dr}{dt}$ *and* $\frac{dh}{dt}$.

$$V = \frac{1}{3}\pi r^2 h$$

Example 4.4. *Sodium chlorate crystals are easy to grow in the shape of cubes by allowing a solution of water and sodium chlorate to evaporate slowly. If* V *is the volume of such a cube with side length* x, *calculate* $\frac{dV}{dx}$ *when* $x = 3$. *What is the meaning of this number?*

Example 4.5. *If the air in a spherical balloon is released so that its volume decreases at a rate of 2* cm^3/sec*, find the rate at which the radius decreases when the radius is 8 cm.*

Example 4.6. *If a spherical ballon is being blown up such that the diameter is increasing at a rate of* 2 *cm/sec, find the rate the surface area is increasing when the surface area is* 36π cm^2*.*

Example 4.7. *A cylinder is growing at a rate of 5* in^3/min *while the radius is staying constant at one foot. Find how fast the height is increasing.*

Example 4.8. *A cone is growing at a rate of 5 in^3/min and the radius is growing at a rate of 4 in/min. Find how fast the height is changing if the radius and height are both 10 in. Is the height increasing or decreasing?*

Example 4.9. *A particle is moving along the curve* $9x^2 - 4y^2 = 36$*. As the particle passes through the point* $(-2\sqrt{2}, 3)$*, its* y*-coordinate increases at the rate of 3 cm/s. How fast is its* y*-coordinate changing at this point?*

Example 4.10. *A particle is moving along the curve* $2^{(x+y)} = x + y + 1$*. As the particle passes through the point* $(1, 0)$*, its* x*-coordinate decreases at the rate of 2 ft/s. How fast is its* y*-coordinate changing at this point? Is the* y*-coordinate increasing or decreasing?*