Test 4. Unit 1 & 2 30%

Unit 3 & 4 70%

— Limit.

* Determinate eg: $\lim_{x\to 2} e^{x^2}$. eg. $\lim_{x\to 0^-} \frac{1}{x} = -\infty$

* Indeterminate. eg: $\frac{0}{0}$, $\frac{\infty}{\infty}$, $0\cdot\infty \left(= \frac{\infty}{\frac{1}{0}}\right)$, $\infty - \infty$

  0^{∞} etc. eg. $\sqrt{x} - \sqrt{x+1}$.

* End Behavior. $f(x)$.

  $\left.\begin{array}{l} \lim_{x\to\infty} f(x), \longrightarrow \text{Right End behavior.} \\ \lim_{x\to -\infty} f(x) \longrightarrow \text{left End behavior} \end{array}\right\}$

— Derivatives. $f(x)$

Def: $\cancel{\lim_{x\to}}$ $f'(x) = \lim_{y\to x} \frac{f(y) - f(x)}{y - x} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$.

$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$

* Table of Derivatives.

$x^n \longrightarrow n\cdot x^{n-1}$

$a^x \longrightarrow (a^x)\cdot \ln a$.

$e^x \longrightarrow e^x$

$\log_a x \longrightarrow \frac{1}{(\ln a)\cdot x}$

$\ln x \longrightarrow \frac{1}{x}$.

$\sin x \longrightarrow \cos x$

$\cos x \longrightarrow -\sin x$

$\tan x \longrightarrow \sec^2 x$

$\sec x \longrightarrow \sec x \tan x$

$\arctan x \longrightarrow \frac{1}{1+x^2}$

$\arcsin x \longrightarrow \frac{1}{\sqrt{1-x^2}}$

$\arccos x \longrightarrow \frac{-1}{\sqrt{1-x^2}}$

* $(c \cdot f(x))' = c \cdot f'(x)$

$(f(x) \pm g(x))' = f'(x) \pm g'(x)$

* Quotient Rule.

$$\left(\frac{f}{g}\right)' = \frac{f'g - f g'}{g^2}$$

* Product Rule v.s Chain Rule.

$(f \cdot g)' = f' g + f \cdot g'$

$[f \circ g(x)]' = [f(g(x))]' = f'(g(x)) \cdot g'(x)$.

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* Implicit Differention.

Exp: $x^2 + xy + y^2 = 1$ find $\frac{dy}{dx}$

$\frac{d}{dx}(x^2 + \underline{xy} + y^2) = \frac{d}{dx}(1)$

$2x + x \cdot y' + y + 2y \cdot y' = 0$

solve for y'.

* Logarithmic Differentiation

Eg: $f(x) = \frac{\sqrt{x} \cdot (x-1)^{19}}{(x+2)^{13}} = y$

$\ln y = \ln\left(\frac{\sqrt{x}\,(x-1)^{19}}{(x+2)^{13}}\right)$

$\ln y = \ln(\sqrt{x}) + \ln(x-1)^{19} - \ln((x+2)^{13}) = \frac{1}{2}\ln x + 19\ln(x-1) - 13\ln(x+2)$

Do Implicit Differentiation.

$\frac{y'}{y} = \underbrace{\frac{1}{2}\frac{1}{x} + 19 \cdot \frac{1}{x-1} - 13\,\frac{1}{x+2}}$ $\Rightarrow$ $y' = (\;y\;) \cdot (\text{———})$

* Inverse Function Differention.

Eg: $\arctan x$. $\rightarrow$ $y = \arctan x$ $\rightarrow$ $x = \tan y$ $\leftarrow \boxed{\tan^2\theta + 1 = \sec^2\theta}$

$1 = \sec^2 y \cdot \frac{dy}{dx}$ $\rightarrow$ $\frac{dy}{dx} = \frac{1}{\sec^2 y} = \frac{1}{1+x^2}$.

* Function Properties. (Application of Derivatives).

— continuous → } I.V.T.
— differentiable. → } M.V.T

— Domain. eg: $\begin{cases} \sqrt{x} \to x \geq 0 \\ \ln x \to x > 0 \\ \frac{\square}{\nabla} \to \nabla \neq 0 \end{cases}$

— Asymptotes. $\begin{cases} \text{vertical} \to \{ f(x) \text{ NOT defined at } a, \ \& \ \lim_{x \to a^+} f(x) = \pm\infty \} \\ \text{horizontal} \to \lim_{x \to \infty} \text{ or } \lim_{x \to -\infty} \\ \text{oblique} \to \frac{p}{q} \to \deg p = \deg q + 1 \to q\overline{)p} \to \text{quotient} = \text{oblique asymptote} \end{cases}$

eg: v.a $\begin{cases} \tan x \to x = \pm\frac{\pi}{2} + 2\pi \\ \ln x \to x = 0 \end{cases}$

eg: h.a $\begin{cases} e^x \to y = 0 \\ \arctan x \to y = \frac{\pi}{2} \ \& \ y = \frac{-\pi}{2} \end{cases}$

eg: o.a $y = \frac{x^2+1}{x+1}$

$x+1\overline{)x^2+1} \to y = x$

— Increasing & Decreasing

↗ if $f'(x) > 0$

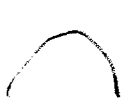
↘ if $f'(x) < 0$.

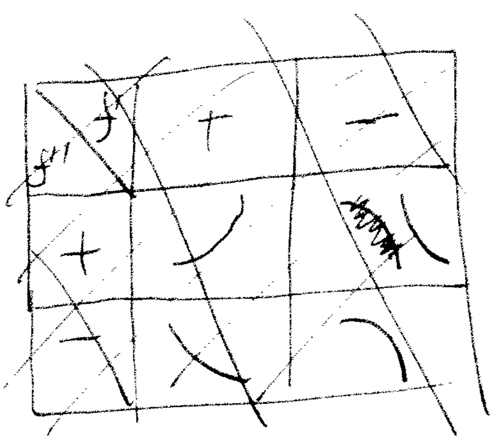
— concave up & concave down

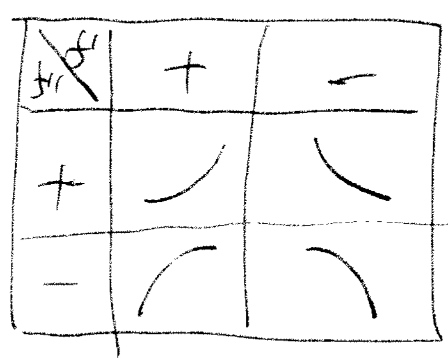
$\begin{cases} \text{concave up if } f''(x) > 0 \\ \text{concave down if } f''(x) < 0 \end{cases}$

— Graph

$y = x^2$ 

$y = -x^2$ 





* Global & Local Extremes.

— critical point.

① in the domain ② $f'(x)=0$ or $f'(x)$ DNE.

— locate local extremes.

△ sign chart of $f'(x)$.

f': $+$ $-$, x → local Max

f': $-$ $+$, x → local Min

$+$ $+$ / $-$ $-$ NOTHING

— Global Extreme

Find local extremes, compare them together with End Points

Exp: $(0, +\infty)$ $\ln x = f(x)$

End Pts. $\underline{\lim_{x\to 0}} f(x) = -\infty$ $\underline{\lim_{x\to\infty}} f(x) = +\infty$

eg: $[0, 1]$ of $f(x)$

$f(0)$. $f(1)$ for End Pts

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* Sketch the graph

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* Newton's Method & Linearization.

- linearization of $f(x)$ at $x=a$

$$L(x) - f(a) = f'(a)(x-a) \to \underline{L(x)} = f(a) + f'(a)(x-a)$$

— Newton's Method.

Guess x_1, $x_2 = x_1 - \cancel{\frac{f(x_1)}{f'}}\ \frac{f(x_1)}{f'(x_1)}$ → $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

* Displacement, Velocity & Acceleration
$s(t)$. $v(t)$. $a(t)$.

$v(t) = s'(t) \longrightarrow s(t) = \int v(t)\,dt$

$a(t) = v'(t) = s''(t) \longrightarrow v(t) = \int a(t)\,dt$.

* Tangent line to the curve $y = f(x)$. @ $\underline{\underline{x=a}} \longrightarrow (a, f(a))$

slope $:= f'(a)$. equation: $y - f(a) = f'(a)(x-a)$

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* Integration Part.

— Riemann Sum & Definite Integral.

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x_i \xleftrightarrow{=} \int_a^b f(x)\,dx.$$

on $[a, b]$

— Left / Right / Mid point Riemann Sum for $n=3$ on $[a\ b]$

— ~~F.T.C.~~ Anti-derivatives.

· $f(x)$ continuous, $F(x)$ is anti-derivative of $f(x)$

if $F'(x) = f(x)$.

$$f(x) \xrightarrow{a.d} F(x) + C$$

— Indefinite Integral of $f(x)$. $\int f(x)\,dx = F(x) + C$

if $F(x)$ is an anti-derivative

of $f(x)$

* Table of anti-derivative

$$\begin{cases} x^n \ (n \neq -1) \longrightarrow \dfrac{x^{n+1}}{n+1} + C \\ x^{-1} = \dfrac{1}{x} \longrightarrow \ln|x| + C \end{cases}$$

$$\begin{cases} a^x \longrightarrow \dfrac{a^x}{\ln a} \\ e^x \longrightarrow e^x \end{cases} \qquad \{\text{trig}\}.$$

* Properties of Definite Integral.

* $\int_a^b c \cdot f(x)\,dx = c \cdot \int_a^b f(x)\,dx$ * $\int_a^b f(x) \pm g(x)\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

* $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ * $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

* F.T.C.

① if $F(x)$ is an anti-derivative of $f(x)$.

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a).$$

② $g(x) = \int_a^x f(t)\,dt$ is an anti-derivative of $f(x)$.

$- g(a) = 0 \qquad - g(b) = \int_a^b f(t)\,dt.$

$- g'(x) = f(x)$ ← Chain Rule. eg. $\int_{x^2}^{2x+1} f(t)\,dt = g(x)$.

what is $g'(x) = ?$

* Substitution : Practice. * Substitution for Definite Integral *

* Areas, & Volumes. { washer
Disk
shell.