Global and Local Extremes

Definitions of Minimum and Maximum Values

Definitions 0.1 *In the definitions below, assume $y = f(x)$ is a function with domain D and c is a number in the domain of f.*

1. *f has an* **absolute maximum** *or* **global maximum** *at $x = c$ if $f(c) \geq f(x)$ for all x in D. $f(c)$ is the* **(absolute) maximum (value)**.
2. *f has an* **absolute minimum** *or* **global minimum** *at $x = c$ if $f(c) \leq f(x)$ for all x in D. $f(c)$ is the* **(absolute) minimum (value)**.
3. *The minimum and maximum values are called the* **(absolute) extreme values** *of f.*
4. *f has an* **local maximum** *or* **relative maximum** *at $x = c$ if $f(c) \geq f(x)$ for x "close enough" to c. $f(c)$ is the* **local maximum (value)**.
5. *f has an* **local minimum** *or* **relative minimum** *at $x = c$ if $f(c) \leq f(x)$ for x "close enough" to c. $f(c)$ is the* **local minimum (value)**.
6. *"close enough" to c means there is an open interval around c where the statement is true. This open interval can be very small.*
7. *The local minimum and local maximum values are called the* **local extreme values** *of f.*

Important Theorems

The Extreme Value Theorem If f is continuous on the closed interval $[a, b]$, then f will attain both a minimum and a maximum in the interval.

In other words, if you consider the interval $[a, b]$ as the domain of f, there will be at least one number c in $[a, b]$ where $f(c)$ is the absolute maximum value, and at least one number d in $[a, b]$ where $f(d)$ is the absolute minimum value.

Fermat's Theorem If f has a local extreme at c, then $f'(c) = 0$ or $f'(c)$ is undefined.

This tells us that the only possible places where f may have a local extreme is where the derivative is equal to 0 or is undefined. Also recall local extremes of f must occur within the domain of the function. A value in the domain of f where the derivative is zero or undefined is called a **critical number** of a function. Note that a critical number does not have to be a local extreme, but a local extreme has to be a critical number.

Find Global Extermes

To find the **absolute** minimum and maximum values of a **continuous** function f on a **closed interval** $[a, b]$:

Step 1. Find the critical point(s) of f in (a, b).

Step 2. Find the function value at all critical point(s) found in step 1.

Step 3. Find $f(a)$ and $f(b)$.

Step 4. The largest value from steps 2 and 3 is the maximum value and the smallest value from steps 2 and 3 is the minimum value.

Find Local Extermes

To find the **local** minimum and maximum values of a **continuous** function f:

Step 1. Find the critical point(s) of f in (a, b).

Step 2. Use the critical point(s) to separate the domain into different areas. Then do the **Sign Chart**

Step 3. From the sign chart in 2, the local maximum occurs on the critical point(s) that changes sign from "$+$" to "$-$".

Step 4. From the sign chart in 2, the local minimum occurs on the critical point(s) that changes sign from "$-$" to "$+$".