Derivative Laws

Basic Derivative Formulas

Here is a list of 1st derivatives of some basic functions.

1. When $f(x) = c$ is the constant functin.

$$\frac{d}{dx}(c) = 0$$

2. When $f(x) = x^n$ is the power functin. Here $n \neq 0$.

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

3. When $f(x) = a^x$ is the exponential functin.

$$\frac{d}{dx}(a^x) = (\ln a)a^x$$

   Especially, when $a = e$, we have:

$$\frac{d}{dx}(e^x) = e^x$$

4. When $f(x) = \log_a x$ is the logarithmic functin.

$$\frac{d}{dx}(\log_a x) = \frac{1}{(\ln a)x}$$

   Especially, when $a = e$, we have:

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

Advanced Derivative Formulas

Of course the basic formulas are limited. The functions we meet in general are much more complicated, but mostly of them are constructed from simple functions by addition, subtraction, product, division and composition. We can find their 1st derivatives by applying the following properties:

Proposition. *Let $f(x)$, $g(x)$ be differentiable functions, let c be a constant. We have:*

1. *Multiplication of a function with a constant*

$$\frac{d}{dx}(cf(x)) = c\frac{d}{dx}f(x) = cf'(x).$$

2. *Addition / subtraction of two functions*

$$\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x) = f'(x) \pm g'(x)$$

3. *Product Rule*

$$\begin{aligned}\frac{d}{dx}[f(x) \cdot g(x)] &= [\frac{d}{dx}f(x)] \cdot g(x) + f(x) \cdot [\frac{d}{dx}g(x)] \\ &= f'(x)g(x) + f(x)g'(x)\end{aligned}$$

4. *Quotient Rule*

$$\begin{aligned}\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) &= \frac{[\frac{d}{dx}f(x)] \cdot g(x) - f(x) \cdot [\frac{d}{dx}g(x)]}{g(x)^2} \\ &= \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}\end{aligned}$$

5. *Chain Rule*

$$\begin{aligned}\frac{d}{dx}[f(g(x))] &= \frac{d}{dx}f(g(x)) \cdot [\frac{d}{dx}g(x)] \\ &= f'(g(x)) \cdot g'(x).\end{aligned}$$

Trigonometry Functions

To start with the derivatives of trigonometry functions, one needs toi know the following limit:

$$\lim_{x\to 0} \frac{sinx}{x} = \lim_{x\to 0} \frac{x}{sinx} = 1$$

And one of the corollary is

$$\lim_{x\to 0} \frac{cos(x) - 1}{x} = 0$$

Thus we can get the 1st derivative of $f(x) = sin(x)$:

Theorem. *The* 1*st derivative of* $sin(x)$ *is:*

$$\frac{d}{dx}(sin(x)) = (sin(x))' = cos(x)$$

Proof. By the definition of 1st derivative, we have:

$$\begin{aligned}
\frac{d}{dx}\left(sin(x)\right) &= \lim_{h\to 0}\frac{sin(x+h)-sin(x)}{h}\\
&= \lim_{h\to 0}\frac{sin(x)cos(h)+sin(h)cos(x)-sin(x)}{h}\\
&= \lim_{h\to 0}\frac{sin(x)(cos(h)-1)+sin(h)cos(x)}{h}\\
&= \lim_{h\to 0}\frac{sin(x)(cos(h)-1)}{h}+\lim_{h\to 0}\frac{sin(h)cos(x)}{h}\\
&= \lim_{h\to 0} sin(x)\frac{(cos(h)-1)}{h}+\lim_{h\to 0} cos(x)\frac{sin(h)}{h}\\
&= sin(x)\cdot 0+cos(x)\cdot 1\\
&= cos(x)
\end{aligned}$$

□

Use similar technique we can see that:

$$\frac{d}{dx}(cos(x)) = (cos(x))' = -sin(x)$$

And hence one can use the Quotient Rule and Product Rule to get the 1st derivatives of $tan(x)$, $cot(x)$, $sec(x)$, $csc(x)$.

Here we give the list of 1st derivatives of trig functions.

1. $$\frac{d}{dx}(sin(x)) = cos(x); \quad \frac{d}{dx}(cos(x)) = -sin(x)$$

2. $$\frac{d}{dx}(tan(x)) = sec^2(x); \quad \frac{d}{dx}(cot(x)) = -csc^2(x)$$

3. $$\frac{d}{dx}(sec(x)) = sec(x)tan(x); \quad \frac{d}{dx}(csc(x)) = -csc(x)cot(x).$$