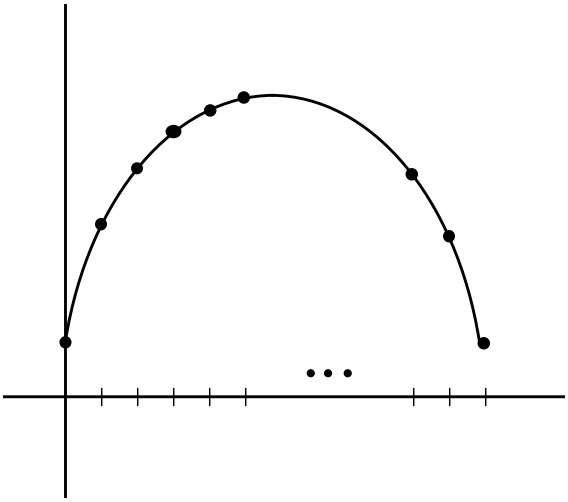
1. 6.5 Average Function Value

The Average Value of f over the interval $[a, b]$ is defined as

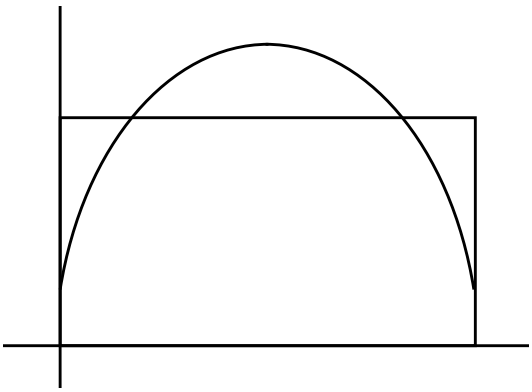
$$f_{ave} = \lim_{n\to\infty} \frac{\sum_{i=1}^{n} [f(x_i^*)]}{n} =$$



2. Mean Value Theorem for Integrals

If f is continuous on $[a, b]$, then there is a number c in $[a, b]$ with

$$\int_a^b f(x)\,dx = f(c)(b-a)$$



3. Examples

Example 3.1. *Find the average value of the function* $f(t) = \sin t$ *over the interval* $[0, \pi]$.

Example 3.2. *Find the average value of the function* $f(x) = \frac{\sin(20x)}{1+\cos^2(20x)}$ *over the interval* $[0, \pi/40]$.

Example 3.3. *Find the average value of the function* $g(x) = x^{1/3}$ *over the interval* $[0, 8]$. *Then find* c *satisfying the conclusion of the Mean Value Theorem for Integrals.*

Example 3.4. *The length of a day, in hours from sunrise to sunset , may be approximated by the formula* $L(t) = 12 + 4\sin(\frac{\pi t}{182})$ *where* t *is the number of days after a Spring Equinox. Find the average length of the day during spring and summer (assume to be* $1/2$ *a year).*