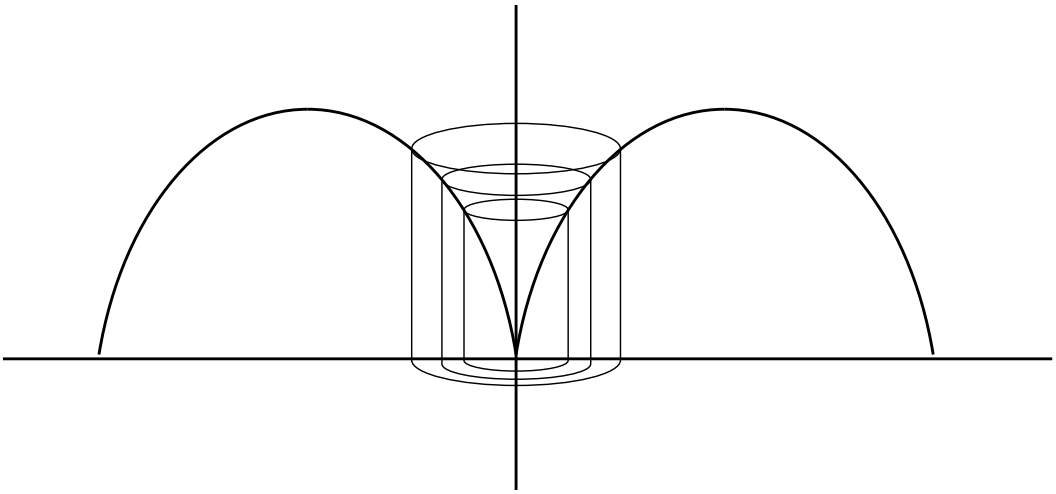
1. 6.3 Volume using Cylindrical Shells

(1) The curve $y = f(x)$ from $x = a$ to $x = b$ rotated about the y-axis will enclose a solid with volume
$$V = \int_a^b 2\pi x f(x)\, dx$$
(2) The curve $x = f(y)$ from $y = c$ to $y = d$ rotated about the x-axis will enclose a solid with volume
$$V = \int_c^d 2\pi y f(y)\, dy$$
(3) To generalize, note that the integrand is the surface area of a cylinder ($2\pi rh$) and everything is in term of the variable for the axis perpendicular to the axis of rotation.

Example 1.1. *Find the volume of the solid obtained by rotating the region about the specified line. $y = x(4 - x)$, $y = x$, about the y-axis*



2. Two Curves

(1) The region bounded on top by $y = f(x)$ and on bottom by $y = g(x)$ from $x = a$ to $x = b$ rotated about the y-axis will enclose a solid with volume

$$V = \int_a^b 2\pi x(f(x) - g(x))\,dx$$

(2) The region bounded on the right by $x = f(y)$ and on the left by $y = g(x)$ from $y = c$ to $y = d$ rotated about the x-axis will enclose a solid with volume

$$V = \int_c^d 2\pi y(f(y) - g(y))\,dy$$

(3) To generalize, note that the integrand is the surface area of a cylinder ($2\pi rh$) and everything is in term of the variable for the axis perpendicular to the axis of rotation.

Example 2.1. *Find the volume of the solid obtained by rotating the region about the specified line.* $y = 3 + 2x - x^2$, $x + y = 3$, *about the y-axis*

3. Lines other than the x and y-axis for the Axis of Rotation

Example 3.1. *Find the volume of the solid obtained by rotating the region about the specified line.* $y = x$, $y = \sqrt{x}$, *about* $x = 2$

Example 3.2. *Set up the integral used to find the volume of the solid obtained by rotating the region about the specified line. Do not evaluate.* $y = e^{-2x}$, $x = 0$, $x = 2$, $y = 0$ *about* $x = 4$

4. Mixed Examples

Example 4.1. *Set up the integral, in terms of* x, *used to find the volume of the solid obtained by rotating the region about the specified line. Do not evaluate.* $y = \ln x$, $y = 0$, $x = e^2$ *about* $x = 1$

Example 4.2. *Set up the integral, in terms of x, used to find the volume of the solid obtained by rotating the region about the specified line. Do not evaluate.* $y = \ln x$, $y = 0$, $x = e^2$ *about* $y = 2$

Example 4.3. *Set up the integral, in terms of y, used to find the volume of the solid obtained by rotating the region about the specified line. Do not evaluate.* $y = \ln x$, $y = 0$, $x = e^2$ *about* $y = 2$