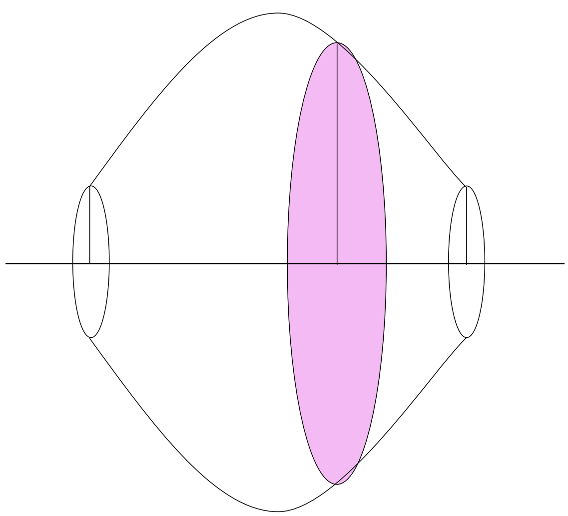
1. 6.2 Solid of Revolution

Definition 1.1. *Let R be a region that lies entirely on one side of a line, L. We may* **revolve** *the region R about the line L to obtain a* **solid of revolution**. *Each point of R is revolved about L so that the point always stays the same distance from L, creating a circle with center on L and radius the distance from L to that point being revolved.*

2. Disc Method



Disc Method:

(1) The curve $y = f(x)$ from $x = a$ to $x = b$ rotated about the x-axis will enclose a solid with cross-sections perpendicular to the x-axis that are discs, so

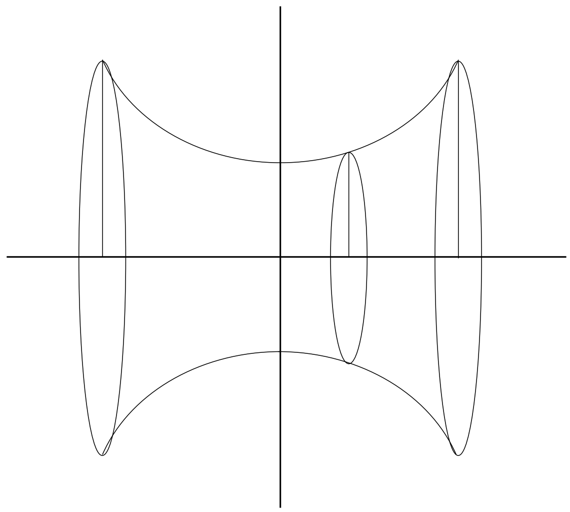
$$V = \int_a^b \pi[f(x)]^2\,dx$$

(2) The curve $x = f(y)$ from $y = c$ to $y = d$ rotated about the y-axis will enclose a solid with cross-sections perpendicular to the y-axis that are discs, so

$$V = \int_c^d \pi[f(y)]^2\,dy$$

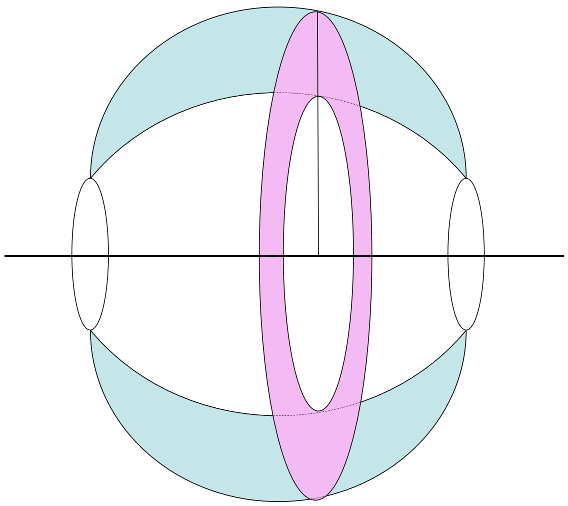
(3) To generalize, note that the integrand is the formula for the area of the circle (πr^2) and the integral is in term of the variable given by the axis of rotation.

Example 2.1. *Find the volume of the solid obtained by rotating the region about the specified line.* $y = \sec x$, $y = 0$, $x = -1$, $x = 1$, *about the* x*-axis*



Example 2.2. *Find the volume of the solid obtained by rotating the region about the specified line.* $y = \sqrt{x}$ $y = 1$ $x = 0$, *about the* y*-axis*

3. Washer Method



(1) The area between the curves $y = f(x)$ and $y = g(x)$ from $x = a$ to $x = b$, where $f(x) \geq g(x)$, rotated about the x-axis will enclose a solid with cross-sections perpendicular to the x-axis that are washers, so

$$V = \int_a^b \pi[f(x)]^2 - \pi[g(x)]^2 \, dx$$

(2) The area between the curves $x = f(y)$ and $x = g(y)$ from $y = c$ to $y = d$, where $f(y) \geq g(y)$, rotated about the y-axis will enclose a solid with cross-sections perpendicular to the y-axis that are washers, so

$$V = \int_c^d \pi[f(y)]^2 - \pi[g(y)]^2 \, dy$$

(3) To generalize, note that the integrand is the formula for the area of the outer circle minus the area of the inner circle ($\pi r_2^2 - \pi r_1^2$) and the integral is in term of the variable given by the axis of rotation.

Example 3.1. *Find the volume of the solid obtained by rotating the region about the specified line.* $y = 2\sec x$, $y = 1$, $x = -\pi/4$, $x = \pi/4$, *about the* x*-axis.*

Example 3.2. *Find the volume of the solid obtained by rotating the region about the specified line.* $y = e^{2x}$, $y = 1$, *and* $x = 2$ *about the* x*-axis.*

Example 3.3. *Set up the integral used to find the volume of the solid obtained by rotating the region about the specified line. Do not evaluate.* $y = e^{2x}$, $y = 1$, *and* $x = 2$ *about the* y*-axis.*

4. Lines other than the x and y-axis for the Axis of Rotation

Example 4.1. *Set up the integral used to find the volume of the solid obtained by rotating the region about the specified line. Do not evaluate.* $y = e^{2x}$, $y = 1$, *and* $x = 2$ *about* $y = -2$.

Example 4.2. *Set up the integral used to find the volume of the solid obtained by rotating the region about the specified line. Do not evaluate.* $y = e^{2x}$, $y = 1$, *and* $x = 2$ *about the line* $x = 4$.

Example 4.3. *Set up the integral used to find the volume of the solid obtained by rotating the region about the specified line. Do not evaluate.* $y = \sec x$, $y = 1$, $x = 1$, *about the line* $y = 1$.

Example 4.4. *Set up the integral used to find the volume of the solid obtained by rotating the region about the specified line. Do not evaluate.* $y = \sec x$, $y = 1$, $x = 1$, *about the line* $y = -1$.

Example 4.5. *Set up the integral used to find the volume of the solid obtained by rotating the region about the specified line. Do not evaluate.* $y = (x-2)^4$, $8x - y = 16$, *about* $x = 10$.