1. 5.5 Substitution

Remark 1.1. *MASTER this method if you are going on to calculus 2. Much of Chapter 7 is based on substitution and then, of course, everything that follows relies on what came before.*

Substitution Rule: Let $u = g(x)$. Recall $\frac{du}{dx} = g'(x)$ and $du = g'(x)dx$. Then

$$\int F'(g(x))g'(x)\,dx = \int F'(u)\,du = F(u) + C = F(g(x)) + C$$

2. To use substitution to integrate:

Assume we want to integrate a function with respect to x *and the previous formulas cannot be applied.*

(1) Look for a composition of two relatively simple functions. Alternatively, look for a factor in a product (or quotient) that appears to be multiplied by its derivative.

(2) Select the "inside" function and call it u (if u was the variable in use and then pick a different letter to use). If there is no obvious composition, look for a function multiplied by something close to its derivative. Your choice for u should be a relatively simple function, but $u = x$ is wasted effort – you are just changing the name of the variable and not the problem.

(3) Find $\frac{du}{dx}$

(4) Then $du = (\frac{du}{dx})dx$, or: $dx = \frac{du}{(\frac{du}{dx})}$.

(5) In the integral substitute in u for the "inside" and substitute $\frac{du}{(\frac{du}{dx})}$ for dx.

(6) Simplify and try to rewrite so the only variable is u.

(7) The goal is to get an integral that you can integrate using formulas you have already learned. If you cannot rewrite so the only variable is u or if the integral is not one you can integrate try a different substitution, more than one substitution may be needed, or reconsider if substitution is needed.

(8) On an indefinite integral, re-substitute back in so the variable is the same one the problem started with. On a definite integral, the original limits of integration are for the original variable. Either re-substitute so you have the original variable or change the limits so they are for the new variable.

3. Examples

Example 3.1. $\int x\sqrt{x^2+4}\,dx$

Example 3.2. $\int \tan x\,dx$

Example 3.3. $\int \frac{x}{(x+2)^3}\,dx$

Example 3.4. $\displaystyle\int \frac{\tan^{-1} x}{x^2+1}\,dx$

4. Definite Integrals

Example 4.1. $\displaystyle\int_0^{\pi/2} e^{\cos 3t} \sin 3t\,dt$

Example 4.2. $\displaystyle\int_{-\pi/3}^{\pi/3} \frac{\sin 2\theta}{\cos^2 2\theta}\,d\theta$

Example 4.3. $\displaystyle\int_e^{e^2} \frac{1}{x \ln x}\, dx$

Example 4.4. $\displaystyle\int \frac{x}{x-3}\, dx$

Example 4.5. $\displaystyle\int_0^1 (1-\sqrt{x})^{50}\, dx$