1. 4.7 Optimization - main steps

- Step 1. Read problem and express all information from the problem mathematically. Use variables to represent any quantity that changes. Numbers may be used for quantities that remain constant.
- Step 2. Find a function for the quantity to be optimized in terms of one variable.
- Step 3. Find the absolute extreme required using the techniques discussed in the notes on finding extrema.
- Step 4. re-read the problem and answer the question.

2. Examples

Example 2.1. A box with an open top has a volume of $16 \, \text{cm}^3$. The base's length is twice its width. Suppose the box is constructed with materials that cost $\$.50/\text{cm}^2$ for the base and $\$.20/\text{cm}^2$ for the material on the sides. Find the dimensions that minimize the cost.

Example 2.2. A right circular cylinder is inscribed in a cone with height H and base radius R. Find the largest possible volume of such a cylinder.

Example 2.3. A right circular cylinder is inscribed in a cone with height H and base radius R. Find the largest possible surface area (without top and bottom) of such a cylinder.

Example 2.4. A boat leaves a dock at noon and travels due south at a speed of 20 km/hr. Another boat has been heading due east at 10 km/hr and reaches the same dock at 2:00 P.M. At what time were the two boats closest together?

Example 2.5. A steel pipe is being carried down a hallway 9 ft wide. At the end of the hall there is a right-angled turn into a narrower hallway 6 ft wide. What is the length of the longest pipe that can be carried horizontally around the corner?

Example 2.6. The upper right-hand corner of a piece of paper, 12 in by 8 in, is folded over to the bottom edge. How would you fold it so as to minimize the length of the fold?

Example 2.7. A painting in an art gallery has height of 3 feet and is hung so that its lower edge is a distance 1 foot above the eye of the observer. How far from the wall should the observer stand to get the best view (i.e. maximize the angle subtended at the observer's eye by the painting)?