RESEARCH STATEMENT

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I am interested in the study of singularities in algebraic geometry, in particular in the computation of characteristic classes and local invariants on singular spaces using intersection theory. I have focused on (generic) determinantal varieties. These are the projective varieties consisting of matrices with corank conditions, and are very classical objects of study in algebraic geometry as they arise naturally in many contexts: quotient singularity, semialgebraic programming, algebraic statistics, etc.

In [21] [24] I obtained explicit formulas for the Chern-Schwartz-MacPherson class, the Chern-Mather class, the local Euler obstructions, the characteristic cycles (of both the variety and the intersection cohomology sheaf) and some other invariants such as Todd class, polar degrees, sectional Euler characteristic and (generic) Euclidean distance degree on determinantal varieties. I also worked on equivariant characteristic classes, and in [22] I obtained explicit formulas for the equivariant Chern-Mather and Chern-Schwartz-MacPherson classes.

In [23] I defined the equivariant (Fulton's) Segre class for equivariant embeddings using Totaro's finite approximation. When the variety X is a hypersurface of some smooth ambient space, I proved that Aluffi's formula [1, Theorem 1.4] can be generalized to the equivariant setting. Using the Schubert 2 package in Macaulay2 ([9]), the formulas I obtained give many examples, and observing them one can find many interesting patterns such as symmetry, positivity and vanishing of certain components of these classes.

My future interest is mainly in two areas. First I would like to compute the Hirzebruch class, the maximum likelihood degree, higher Euler obstructions, and some other Chern classes for determinantal varieties, and see how they depend on the natural stratification. Secondly I am very interested in understanding how to compute local invariants and equivariant characteristic classes of quotient singularities via tools from representation theory, and carrying out such computations to interesting spaces. I am also interested in computing the local invariants of other spaces such as ladder determinantal varieties and Schubert varieties.

1. Overview

Recall that the Poincaré -Hopf theorem says that on a smooth variety X, the integration of the total Chern class equals the Euler characteristic. A generalization of the total Chern class to possible singular spaces was conjectured by Grothendieck and Deligne in the 1970s, and was constructed by R.D. MacPherson over $\mathbb{C}$ in 1974 ([14]). This generalization is a class in the Chow group $A_*(X)$ that integrates to the Euler characteristic and agrees with the total Chern class when X is smooth. It corresponds by the Alexander isomorphism to the class defined by M.-H. Schwartz in [17], and is called the Chern-Schwartz-MacPherson class of X, denoted by $c_{SM}(X)$. In 1990 G. Kennedy generalized MacPherson's result from $\mathbb{C}$ to arbitrary algebraically closed field of characteristic 0 ([13]). The c_{SM} class is a linear combination of classes of different dimensions, and contains rich information about the space. It has been the object of intense study, that has been computed for many spaces such as toric varieties, Schubert cells, etc. It is conjectured that these classes are effective in many cases.

Two key ingredients used in the construction of Chern-Schwartz-MacPherson class in [14] are the local Euler obstruction Eu_X and the Chern-Mather class $c_M(X)$. The local Euler obstruction is an integer valued function that can be viewed as the Euler characteristic of local complex link space. It was originally defined over $\mathbb{C}$ by MacPherson using obstruction theory, and was generalized to arbitrary algebraically closed field using the Nash Blowup and Segre classes by González-Sprinberg and Verdier. The Chern-Mather class is a class in the Chow group $A_*(X)$, defined as the pushforward of the Chern class of the Nash tangent bundle from the Nash Blowup of X. When X is smooth, the Chern-Mather class agrees with the total Chern class of X. The local Euler obstruction and the Chern-Mather class are defined over any algebraically closed field, while the Chern-Schwartz-MacPherson class is only defined over characteristic 0 fields. They are both very important concepts and have been computed for many spaces.

In [24] I gave a projective bundle description of the Nash Blowup of a determinantal variety $\tau_{m,n,k}$, and used intersection theory to explicitly compute the local Euler obstruction $Eu_{\tau_{m,n,k}}$. The formula over $\mathbb{C}$ was obtained using topology method by T. Gaffney et al. Using this formula I then proved that the Chern-Mather class of a determinantal variety can be computed via the 'Tjurina transform', which is a small resolution and is identified to a projective bundle over a Grassmannian. Using the Chow ring structure of the projetive bundle I then in [21] obtained an explicit formula for the Chern-Mather class of $\tau_{m,n,k}$. In particular, when the base field is of characteristic 0, the relation among the Euler obstructions and the indicator functions along the strata induces a formula for the Chern-Schwartz-MacPherson class of $\tau_{m,n,k}$. Recall that for a projective variety, knowing the Chern classes is essentially equivalent to knowing the class of the characteristic cycle, which can be used to compute other characteristic classes and local invariants. Thus for determinantal varieties I also obtained formulas and numerical examples for the polar classes, the Todd class, the (generic) Euclidean distance degree and the sectional Euler characteristic. On the basis of explicit computations in low dimensions, I observed some interesting patterns concerning symmetry, vanishing and effectiveness of Chern classes. I formulated precise conjectures concerning the effectiveness and the vanishing of specific terms in the Chern-Schwartz-MacPherson classes of the largest strata $\tau_{m,n,k} \smallsetminus \tau_{m,n,k+1}$.

When the base field is $\mathbb{C}$, I proved that the characteristic cycle assigned to the Intersection Cohomology Sheaf on a determinantal variety equals its conormal cycle, and hence is irreducible. This indicates that on determinantal varieties the Intersection Cohomology Sheaf is mapped to the local Euler obstruction under the canonical map from the Grothendieck group of constructible sheaves to the group of constructible functions. This irreducibility is an interesting and rare phenomenon, and it implies that the Chern-Mather class of the space equals the pushforward of the total Chern class from any small resolution. It is known to be true for some special varieties such as certain Schubert varieties in flag manifolds [12] and theta divisors of Jacobians [4]. An reducibility example can be found in [2].

When X is a complex G-variety, T. Ohmoto generalized the theory of Chern-Mather class and Chern-Schwartz-MacPherson class to an equivariant setting in [16]. In [22] I lifted the natural $\mathbb{C}^*$ action on a determinantal variety equivariantly to its Tjurina transform, and used Berline-Vergne's localization formula to give formulas on the equivariant Chern-Mather class and the equivariant Chern-Schwartz-MacPherson class of the determinantal variety. The formulas I obtained are very explicit and seem to have close connections with the weight functions in the representation of the torus.

When X is a hypersurface of a smooth ambient space, the Chern-Schwartz-MacPherson class of X can be computed via Fulton's Segre class using Aluffi's formula ([1, Theorem 1.4]). In [23] I defined the equivariant Segre classes for equivariant closed embeddings. When X is a complex variety with a $\mathbb{C}^*$ action, I proved that Aluffi's formula can be generalized to the equivariant setting.

2. Future Work

2.1. Motivic Chern Class and Hirzebruch Class. (This is a joint project with Xia Liao.)

On a smooth complex algebraic variety X, the Chern-Schwartz-MacPherson class, the Todd class and the L class are unified by the Hirzebruch Class (or the modified Todd class) $T_{y*}(TX)$: they correspond to the y value $-1, 0, 1$ respectively. For singular spaces, the Hirzebruch Class T_{y*} is defined as a group homomorphism from the relative Grothendieck group $K_0(\mathcal{VAR}/X)$ to the tensored homology group $H_*(X)\otimes\mathbb{Q}[y]$, which is the composition of two maps $T_{y*} = td_{(1+y)}\circ mC_*$. Here the motivic Chern class $mC_*\colon K_0(VAR/X) \to G_0(X)\otimes\mathbb{Z}[y]$ is a group homomorphism to the tensored group of coherent sheaves, and $td_{(1+y)}$ is a group homomorphism $G_0(X)\otimes\mathbb{Z}[y] \to H_*(X)\otimes\mathbb{Q}[y]$.

When X admits a resolution $p\colon \hat{X} \to X$ such that $p^{-1}(X_{sing})$ is a simple normal crossing divisor, as shown in [20][15] both the motivic Chern class and the Hirzebruch class have explicit formulas in terms of the sheaves of logarithmic poles. Our project is to study the sheaf of logarithmic poles on such resolutions over determinantal varieties $\tau_{m,n,k}$ and use the recursive relation obtained by the pushforward to compute the motivic Chern class and the Hirzebruch class . Also, an alternative definition of the Hirzebruch class uses Saito's Mixed Hodge Module theory([3]), which is quite mysterious and unclear for determinantal varieties. We hope that the computation of the Hirzebruch classes and their decompositions along the stratification can reveal some properties on the Mixed Hodge Module over determinantal varieties.

In the paper [3] the authors mentioned that for some singular spaces $mC_0([\mathcal{O}_X]) \neq [\mathcal{O}_X]$, $T_{0*}([\mathrm{Id}_X]) \neq td_*(X)$ and $T_{1*}(\mathrm{Id}_X) \neq L_*(X) := L_*(\mathcal{IC}^\bullet_X)$. We wish to use the computation to see whether these discrepancies hold for determinantal varieties.

2.2. Representation Theory. The McKay Correspondence connects the representation of a finite group G on an ambient space M and the singularity of the quotient M/G. As shown in [5] for quotient singularities $\mathbb{C}^n/G$ the Hirzebruch class coincides with the Molien series of G under suitable substitution of variables, and the Hirzebruch has certain positivity properties . Also, the formula I obtained for the equivariant Chern-Mather class of a determinantal variety looks very similar to the weight functions used in [6], in which Feher and Rimanyi compute the equivariant Chern classes of matrix Schubert cells using methods from representation theory.

I am interested in understanding how the local invariants/characteristic classes of the quotient singularity are computed via studying the group action of the ambient space, and possible approaches to the positivity of characteristic classes via representation theory.

2.3. Maximum Likelihood Degree and other Chern Classes. Another interesting invariant of a stratified complex variety X is the maximum likelihood degree. The index formula shows that it is closely related with the Euler obstructions along the strata and the Euler characteristics of special hyperplane arrangements, i.e., the very affine part X^{aff}. For determinantal varieties $\tau_{m,n,k}$, when $k = n-2$ the formula was conjectured by June Huh and Bernd Sturmfels in [10], and was proved by Botong Wang and Jose Israel Rodriguez in [11]. I am interested in modifying their method and finding a formula for the maximum likelihood degree of $\tau_{m,n,k}$ when $k < n-2$.

2.4. Higher Euler Obstruction and Characteristic Cycles. The irreducibility of the characteristic cycle is equivalent to the vanishing of the microlocal multiplicities, which are defined via higher Euler obstructions. These obstructions are defined as the sum over the flags of strata of products of the Euler characteristic of higher complex link spaces. These are the local intersections of nearby neighborhoods of the variety with generic linear spaces. For determinantal varieties I would like to compute the higher Euler obstructions and explain the vanishing of the microlocal multiplicities via the mutual effect of the local intersections along the strata, and compute the characteristic cycles of other constructible sheaves.

2.5. **Ladder Determinantal Varieties.** I am interested in how to compute the Chern class and local invariants (Euler obstruction, maximum likelihood degree, etc) of more general determinantal varieties, in particular the ladder determinantal varieties. Locally the ladder determinantal varieties can be viewed as opposite cells of general Schubert varieties, thus knowing the local Euler obstruction of ladder determinantal varieties will imply a formula for the local Euler obstruction of general Schubert cells.

2.6. **Other Chern Classes.** The virtual Chern class $c_*(X)$ is another generalization of the total Chern class. It is defined using the Segre class of the embedding into an ambient space. Using my definition of equivariant Segre class we can also define its equivariant version. I am interested in computing both the ordinary and the equivariant virtual Chern class for determinantal varieties.

Recently, a new canonical Chern class ($c_\theta(X)$) has been introduced by James Fullwood and has been proved to be the same as the Chern-Schwartz-MacPherson class for hypersurfaces. It would be interesting to check whether the classes match for varieties of higher codimensions. I am interested in computing such Chern classes for some simple varieties like toric varieties and determinantal varieties, and comparing the results with their Chern-Schwartz-MacPheson classes.

2.7. **Applications.** There are methods to apply the (algebraic) geometry properties of the determinantal varieties to convex/non-convex optimization problems (semi-algebraic programming for example). I am interested in understanding the these applications and exploring whether there are any problems I can solve with intersection theory.

3. More Details of My Work

3.1. **Local Euler Obstructions.** Let $\tau_{m,n,k}$ be the determinantal variety over an arbitrary algebraically closed field K. I identified the Nash Blowup of $\tau_{m,n,k}$ as the projective bundle $\mathbb{P}(Q_1^\vee \otimes S_2)$ over $G(k,n)\times G(n-k,m)$, which is smooth. Here (S_1,Q_1) and (S_2,Q_2) are the universal subbundles and quotient bundles over $G(k,n)$ and $G(n-k,m)$ respectively. By computing certain Chern classes and Segre classes in [24] I proved the following theorems.

Theorem 3.1 (Euler Obstructions)**.** *For any $\varphi \in \tau^\circ_{m,n,k+i} := \tau_{m,n,i} \smallsetminus \tau_{m,n,i+1}$ of co-rank exactly i, the local Euler obstruction of $\tau_{m,n,k}$ at φ is:*

$$Eu_{\tau_{m,n,k}}(\varphi) = \int_{G(k,k+i)\times G(i,m-n+k+i)} c_{top}(S_1^\vee \otimes S_2)c_{top}(Q_1^\vee \otimes Q_2).$$

When one reduces the size of the matrix while preserving its co-rank, the theorem shows that the local Euler obstruction stays unchanged. Thus it is enough to compute the local Euler obstruction on the smallest stratum $\tau_{m,n,n-1}$. Define $e(m,n,k) := Eu_{\tau_{m,n,k}}(\varphi)$ for $\varphi \in \tau_{m,n,n-1}$. By explicitly calculating the top Chern class of certain bundles on Grassmannians I proved the following Pascal's relation.

Theorem 3.2 (Pascal's Triangle)**.** *For $k = 0, 1, \cdots, n-2$ we have the following relation:*

$$e(m,n,k) + e(m,n,k+1) = e(m+1,n+1,k+1).$$

Thus we have $e(m,n,k) = \binom{n-1}{k}$. *In particular when* $\varphi \in \tau^{\circ}_{m,n,k+i}$ *has co-rank exactly* $k+i$, *we have*

$$Eu_{\tau_{m,n,k}}(\varphi) = \binom{k+i}{i}.$$

When the base field is $\mathbb{C}$, the formula for the local Euler obstruction of determinantal varieties was obtained by T. Gaffney, N. Grulha and M. Ruas in [7] using a different (topological) method.

3.2. **Characteristic classes.** Using the local Euler obstruction formula above I proved that the Chern-Mather class of $\tau_{m,n,k}$ equals the pushforward of the total Chern class from its Tjurina transform $\hat{\tau}_{m,n,k}$. The variety $\hat{\tau}_{m,n,k}$ is a small resolution of $\tau_{m,n,k}$, and can be identified with the smooth projective bundle $\mathbb{P}(Q^{\vee m})$ over $G(k,n)$. Here Q is the universal quotient bundle. Thus the class $c_M(\tau_{m,n,k})$ can be computed by intersection theory in the Grassmannian $G(k,n)$.

Recall that the Chow group of $\mathbb{P}^{mn-1}$ may be realized as $\mathbb{Z}[H]/(H^{mn})$, where H is the hyperplane class $c_1(\mathcal{O}(1))\cap[\mathbb{P}^{mn-1}]$. The Chern-Mather class $c_M(\tau_{m,n,k})$ then admits the form of a polynomial in H. For $k\geq 1$, $i,p = 0,1\cdots m(n-k)$, we define the following integers

$$\begin{aligned} A_{i,p}(k) = A_{i,p}(m,n,k) &:= \int_{G(k,n)} c(S^{\vee}\otimes Q)c_i(Q^{\vee m})c_{p-i}(S^{\vee m})\cap[G(k,n)] \\ B_{i,p}(k) = B_{i,p}(m,n,k) &:= \binom{m(n-k)-p}{i-p}; \end{aligned}$$

and let $A(m,n,k) = \left[A_{i,p}(k)\right]_{i,p}$; $B(m,n,k) = \left[B_{i,p}(k)\right]_{i,p}$; $\mathcal{H}(m,n,k) = [H^{mk+p-i}]_{ip}$ be $m(n-k)+1\times m(n-k)+1$ matrices. Here we assume $\binom{a}{b} = 0$ for $a<b$ or $a<0$ or $b<0$. Then in [21] I proved the following theorems.

Theorem 3.3 (Main Formula I). *When* $k=0$, $\tau_{m,n,0} = \mathbb{P}^{mn-1}$ *is the projective space, and we have* $c_M(\mathbb{P}^{mn-1}) = (1+H)^{mn}$. *For* $k\geq 1$, *the Chern-Mather class of* $\tau_{m,n,k}$ *is given by*

$$c_M(\tau_{m,n,k}) = \operatorname{trace}(A(m,n,k)\cdot\mathcal{H}(m,n,k)\cdot B(m,n,k)).$$

In particular, when K is of characteristic 0, we can compute the Chern-Schwartz-MacPherson class of $\tau_{m,n,k}$ using the relation among the indicator functions and the local Euler obstruction functions of the strata.

Corollary 3.4 (Main Formula II). *The Chern-Schwartz-MacPherson class of* $\tau_{m,n,k}$ *is given by:*

$$c_{SM}(\tau_{m,n,k}) = \sum_{i=0}^{n-1-k}(-1)^i\binom{k+i-1}{k-1}c_M(\tau_{m,n,k+i}).$$

In terms of Characteristic cycles we have the following result.

Proposition 3.5. *Let* $c_M(\tau_{m,n,k}) = \sum_{l=0}^{mn-1}\beta_l H^{mn-1-l}$ *be the Chern-Mather class of* $\tau_{m,n,k}$. *Then the classes of the projectivized conormal cycle* $Con(\tau_{m,n,k})$ *and the characteristic cycle* $Ch(\tau_{m,n,k})$ *are given by*

$$\begin{aligned} Con(\tau_{m,n,k}) =& (-1)^{(m+k)(n-k)-1}\sum_{j=1}^{mn-2}\sum_{l=j-1}^{mn-2}(-1)^l\beta_l\binom{l+1}{j}h_1^{mn-j}h_2^j\cap[\mathbb{P}^{mn-1}\times\mathbb{P}^{mn-1}]; \\ Ch(\tau_{m,n,k}) =& \sum_{i=0}^{n-1-k}(-1)^i\binom{k+i-1}{k-1}Con(\tau_{m,n,k+i}). \end{aligned}$$

For projective varieties, the coefficients of the (projectivized) conormal cycle are closely related to other interesting invariants such as polar degrees, (generic) Euclidean distance degree and sectional Euler characteristics. I also obtained formulas for those invariants for determinantal varieties.

3.3. **Characteristic Cycle of Intersection Cohomology Sheaf.** When the base field is $\mathbb{C}$, the open cells $\{\tau^\circ_{m,n,i} | i = 0, 1, \cdots, n-1\}$ form a Whitney stratification of the projective space $\mathbb{P}^{mn-1} = \tau_{m,n,0}$. The characteristic cycle of the intersection cohomology sheaf of a determinantal variety $\tau_{m,n,k}$ is a linear combination of the conormal cycles: $\mathcal{CC}(\mathcal{IC}^\bullet_{\tau_{m,n,k}}) = \sum_{i=0}^k r_i(\mathcal{IC}^\bullet_{\tau_{m,n,k}})[T^*_{\tau_{m,n,i}}\mathbb{P}^{mn-1}]$. The coefficients $r_i(\mathcal{IC}^\bullet_{\tau_{m,n,k}})$ are called microlocal multiplicities, and the Kashiwara microlocal index formula shows that for any point $\varphi \in \tau_{m,n,k}$, the following equation holds: $\chi_\varphi(\mathcal{IC}^\bullet_{\tau_{m,n,k}}) = \sum_{i=0}^k Eu_{\tau_{m,n,i}}(\varphi) r_i(\mathcal{IC}^\bullet_{\tau_{m,n,k}})$. Also, notice that the Tjurina transform $\nu\colon \hat{\tau}_{m,n,k} \to \tau_{m,n,k}$ is a small resolution, thus from [8, §6.2] we have $\chi_\varphi(\mathcal{IC}^\bullet_{\tau_{m,n,k}}) = \chi(\nu^{-1}(\varphi))$. We then obtain the following result in [21].

Theorem 3.6. *For $i = 0, 1, \cdots, n-1$, let $r_i := r_i(\mathcal{IC}^\bullet_{\tau_{m,n,k}})$ be the microlocal multiplicities. Then we have $r_i = 1$ when $i = k$, and $r_i = 0$ otherwise. Thus the characteristic cycle of its intersection cohomology sheaf $\mathcal{CC}(\mathcal{IC}^\bullet_{\tau_{m,n,k}})$ equals the conormal cycle $[T^*_{\tau_{m,n,k}}\mathbb{P}^{mn-1}]$, and is irreducible.*

If the characteristic cycle of the intersection cohomology sheaf on a complex variety X is irreducible, for any small resolution $p\colon Y \to X$ the Chern-Mather class $c_M(X)$ equals the pushforward of the total Chern class $p_*(c_{SM}(Y))$. However, as pointed out in [12, Rmk 3.2.2], this is a rather unusual phenomenon. It is known to be true for Schubert varieties in a Grassmannian, for certain Schubert varieties in flag manifolds [12], and for theta divisors of Jacobians [4]. For a discussion of resucibility cases, see [2].

3.4. **Equivariant Characteristic Classes.** Let $T = (\mathbb{C}^*)^{m+n}$ be a complex torus acting on $\tau_{m,n,0} = \mathbb{P}^{mn-1}$ by $(s_1, s_2, \cdots s_m; t_1, t_2, \cdots t_n) \times (a_{ij})_{m\times n} \mapsto (s_i^{-1} a_{i,j} t_j)_{m\times n}$, and the subvarieties $\tau_{m,n,k}$ are invariant under this action. The equivariant Chern-Mather class and the equivariant Chern-Schwartz-MacPherson class of $\tau_{m,n,k}$ are viewed as classes in the equivariant Chow group $A^T_*(\mathbb{P}^{mn-1}) = \Lambda_T[\zeta]/\prod_{i,j}(\zeta + t_i - s_j)$. Here $\Lambda_T = \mathbb{Z}[s_1 \cdots s_m; t_1 \cdots t_n]$ is the equivariant Chow group of a point, and $\zeta = c_1^T(\mathcal{O}(1)) \cap [\mathbb{P}^{mn-1}]_T$ is the equivariant hyperplane class of $\mathbb{P}^{mn-1}$.

I first lifted the T action equivariantly to the Tjurina transform $\hat{\tau}_{m,n,k}$, the equivariant Chern-Mather class $c^T_M(\tau_{m,n,k})$ then equals the (equivariant) pushforward of the equivariant Chern-Schwartz-MacPherson class of $\hat{\tau}_{m,n,k}$. To compute $c^T_{SM}(\hat{\tau}_{m,n,k})$, I used the Berline-Vergne's localization formula ([19]), which says that equivariant homology classes can be reconstructed by local contributions. Thus, by explicitly calculating the local weight decompositions around the fixed points I obtained the following result in [22].

Theorem 3.7. *Define the functions $\gamma_h(m,n,k) \in \Lambda_T$ as*

$$\gamma_h(m,n,k) = \sum_{\substack{(i,j)\\ \{i_1,i_2\cdots i_k\}\\ j\notin\{i_1,i_2\cdots i_k\}}} (t_i - s_i)^{mn-1-h} \prod_{\substack{\alpha=1,2\cdots m\\ \beta=1,2\cdots n\\ \beta\notin\{i_1,i_2\cdots i_k\}\\ (\alpha,\beta)\neq(i,j)}} \frac{1 + s_\alpha - t_\beta + t_j - s_i}{s_\alpha - t_\beta + t_j - s_i} \prod_{\substack{\alpha=1,2\cdots n\\ \beta=1,2\cdots k\\ \alpha\notin\{i_1,i_2\cdots i_k\}}} \frac{1 + t_\alpha - t_{i_\beta}}{t_\alpha - t_{i_\beta}}.$$

Then for $k \geq 1$, the equivariant Chern-Mather class of $\tau_{m,n,k}$ and the equivariant Chern-Schwartz-MacPherson class of $\tau_{m,n,k}$ are given by

$$c^T_M(\tau_{m,n,k}) = \sum_{h=0}^{mn-1} \gamma_h(m,n,k)\zeta^h$$

$$c^T_{SM}(\tau_{m,n,k}) = \sum_{h=0}^{mn-1} \sum_{i=0}^{n-k-1} (-1)^i \binom{k-1+i}{i} \gamma_h(m,n,k+i)\zeta^h.$$

For $k = 0$, $c^T_M(\tau_{m,n,0}) = c^T_{SM}(\tau_{m,n,0}) = c^T_{SM}(\mathbb{P}^{mn-1}) = (1+\zeta)^{mn}$.

3.5. **Equivariant Chern-Schwartz-MacPherson Classes of Hypersurfaces.** Let G be a reductive subgroup of the general linear group, and let $Y \subset X$ be an equivariant closed embedding of complex G varieties. Let $U_i \subset V_i$ be the l-dimensional representation of G used in [18] [16]. I define the equivariant Segre class to be

$$s_i^G(Y, X) := s_i(U_i \times_G Y, U_i \times_G X); \quad s^G(Y, X) := \oplus s_i^G(Y, X)$$

for some U_i such that the codimension of $(V_i \smallsetminus U_i, V_i)$ is sufficiently large. When $Y \subset X$ are smooth the above definition agrees with the equivariant Segre-Schwartz-MacPherson class defined in [16]. Then in [23] I proved the following result.

Theorem 3.8. *Let $X \subset M$ be a hypersurface in a smooth ambient space, and let $Y \subset X$ be the singular subvariety. We have the following formula:*

$$c_{SM}^G(X) = c^G(TM) \cap (s^G(X, M) + c^G(\mathcal{O}(X))^{-1} \cap (s^G(Y, M)^\vee \otimes_{\mathcal{O}_M} \mathcal{O}(X))).$$

When G is the trivial group, this is Aluffi's formula [1, Theorem 1.4].

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