2.3. Continuity

Definition 2.3.1 (Intuitive idea used in algebra based on graphing). *A function, f, is continuous on the interval (a, b) if the graph of $y = f(x)$ can be drawn over the interval (a, b) without lifting your pencil.*

Definition 2.3.2 (The carefully thought-out calculus version based on limits).

1. *A function, f, is* **continuous** *at $x = a$ if* ______________________
2. *A function, f, is* **continuous on the interval** (a, b) *if f is continuous at every value in (a, b).*
3. *A function, f, is* **left continuous (or continuous from the left)** *at $x = a$*

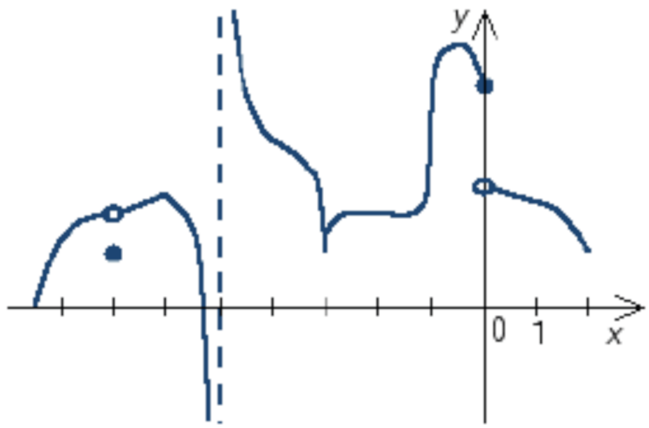
   if ______________________
4. *A function, f, is* **right continuous (or continuous from the right)** *at*

   $x = a$ if ______________________
5. *A function, f, is* **continuous on the interval** $[a, b]$ *if f is continuous at every value in (a, b), f is right continuous at a and f is left continuous at b.*
6. *A function, f, is* **discontinuous** *at $x = a$ if* ______________________

Remarks 2.3.1.

1. *Use what you know about functions from algebra. Polynomials are continuous everywhere and rational functions are discontinuous when the denominator is 0.*
2. *If a is NOT is the domain of f then f is CANNOT be continuous at that value.*
3. *Usually, one looks for the domain and discontinuities to determine where a function is continuous.*

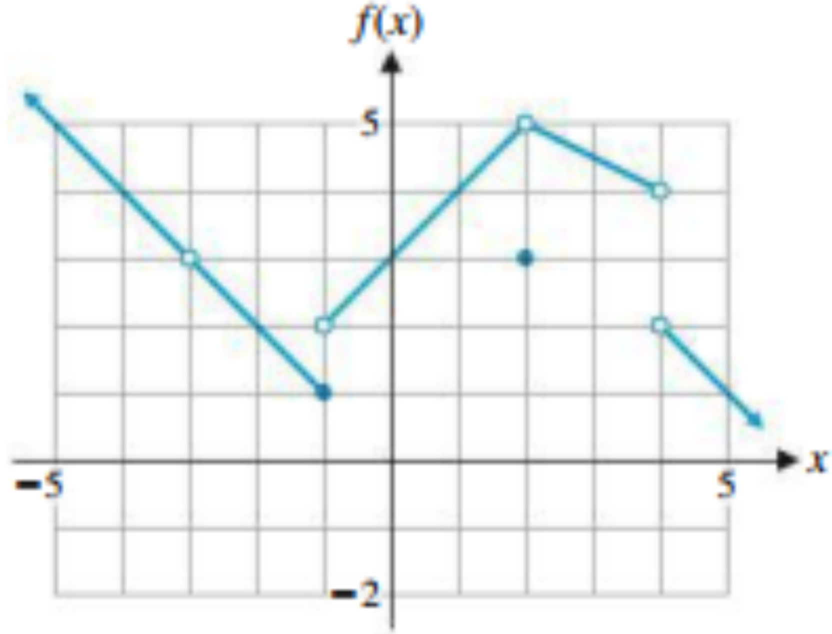
Example 2.3.1. *Determine the discontinuities and the intervals on which f is continuous.*



Example 2.3.2. *Where is* $f(x) = \dfrac{x+1}{x^2 - x - 2}$ *continuous?*

(1) None of these
(2) $(-\infty, \infty)$
(3) $(-\infty, 2) \cup (2, \infty)$
(4) $(-\infty, -1) \cup (-1, \infty)$
(5) $(-\infty, -1) \cup (-1, 2) \cup (2, \infty)$

Example 2.3.3. *Evaluate the problems below given the graph of $f(x)$:*



(1) $f(2) =$

(2) $f(4) =$

(3) $f(-1) =$

(4) $f(-3) =$

(5) $\lim_{x \to 2^-} f(x) =$

(6) $\lim_{x \to 2^+} f(x) =$

(7) $\lim_{x \to 2} f(x) =$

(8) $\lim_{x \to 4^-} f(x) =$

(9) $\lim_{x \to 4^+} f(x) =$

(10) $\lim_{x \to 4} f(x) =$

(11) $\lim_{x \to -3} f(x) =$

(12) $\lim_{x \to -1^-} f(x) =$

(13) $\lim_{x \to -1^+} f(x) =$

(14) $\lim_{x \to -1} f(x) =$