7.2. Partial Derivatives

Definition 7.2.1. *Let* $z = f(x, y)$ *be a function of two variables. We define the* **partial derivative of** f **with respect to** x *as*

and the **partial derivative of** f **with respect to** y *as*

Remark 7.2.1. *Some common notations are*

Example 7.2.1. *Find* $f_x(x, y)$ *and* $\frac{\partial z}{\partial y}$ *if* $z = f(x, y) = 5x^3y - 2xy^2 + 7x - 4y + 6$.

Example 7.2.2. *Find* $f_x(x, y)$ *and* $f_y(x, y)$ *if* $z = f(x, y) = xe^y + y \ln x + \frac{x}{y}$.

Example 7.2.3. *Find* $f_x(x, y)$ *and* $f_y(x, y)$ *if* $z = f(x, y) = e^{2x^2 - xy + y^2}$.

Example 7.2.4. *Find* $f_x(x, y)$ *and* $f_y(x, y)$ *if* $z = f(x, y) = x^2 \ln(y^3 + xy)$.

Example 7.2.5. *Find* $R_p(p, q)$ *and* $R_q(p, q)$ *if* $R(p, q) = 12p - 4q + 4pq - p^4 + q^3$.

Second-order partial derivatives

Definition 7.2.2. *Let $z = f(x, y)$ be a function of two variables. We define the* **second order partial derivative of** f *are*

(1)

(2)

(3)

(4)

Example 7.2.6. *Find $f_{xx}(x, y)$, $\frac{\partial^2 z}{\partial y^2}$, $f_{xy}(x, y)$, and $f_{yx}(x, y)$ if $z = f(x, y) = 5x^3y - 2xy^2 + 7x - 4y + 6$.*

Example 7.2.7. *Find* $f_{yx}(x, y)$ *if* $z = f(x, y) = xe^y + y \ln x + \frac{x}{y}$.

Example 7.2.8. *Find* $f_{xx}(x, y)$ *if* $z = f(x, y) = e^{2x^2 - xy + y^2}$.

Example 7.2.9. *Find* $f_{xy}(x, y)$ *if* $z = f(x, y) = x^2 \ln(y^3 + xy)$.

Example 7.2.10. *Find $R_{qp}(p,q)$ and $R_{qq}(p,q)$ if $R(p,q) = 12p - 4q + 4pq - p^4 + q^3$.*

Example 7.2.11. *For $C(x,y) = 3x^2 + 10xy - 8y^2 + 4x - 15y - 120$, find $C_{yy}(3,-2)$.*

Applications

Example 7.2.12. *A company manufactures two types of calculators, A and B. The weekly price-demand equations and cost equations are*

$p = 15 - 2x + y$

$q = 20 + x - 2y$

$C(x,y) = 20 - 2x + y$

where p is the unit price of A, q is the unit price of B, x is the weekly demand for A, y is the weekly demand for B, and $C(x,y)$ is the cost function.

(1) Find the profit function $P(x,y)$ (in thousands of dollars).

(2) Find $P_x(2,4)$

(3) Find x *and* y *such that* $P_x(x,y)=0$ *and* $P_y(x,y)=0$ *simultaneously.*