

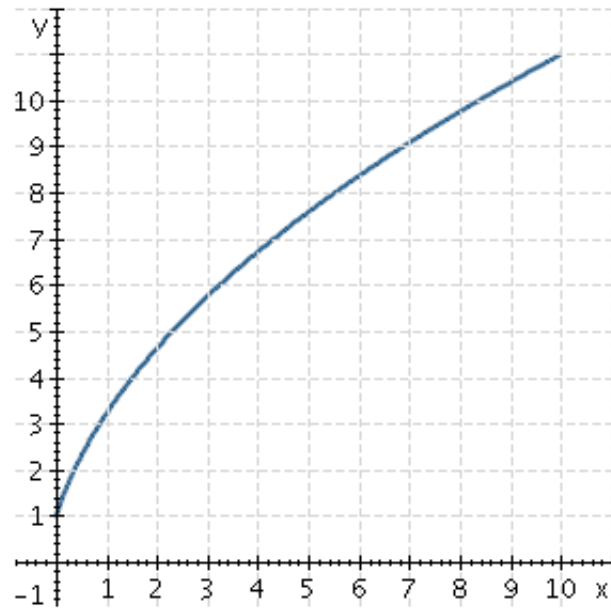
5.4. THE DEFINITE INTEGRAL

We assume for this section that f is a continuous function on the interval $[a, b]$ and n is a positive integer.

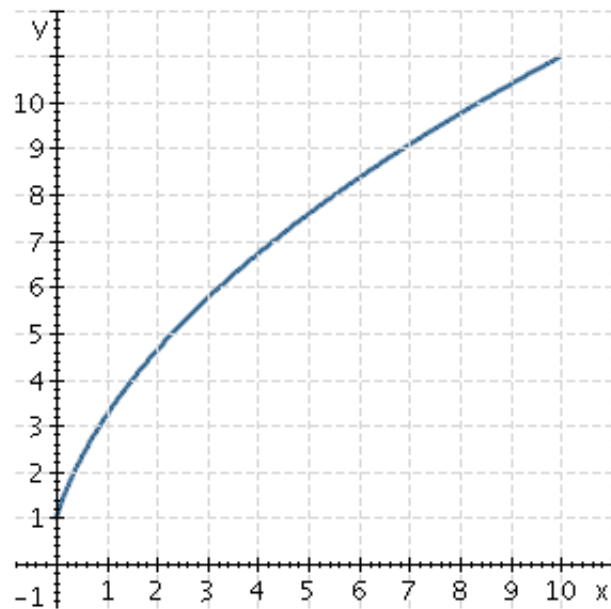
- (1) We may subdivide the interval $[a, b]$ into n subintervals. Denote the endpoints of the subintervals $x_0, x_1, x_2, \dots, x_n$ where $a = x_0$, $b = x_n$ and $x_{i-1} < x_i$. This is called a _____ of the interval $[a, b]$.
- (2) It is most common to choose the subintervals to all have the same width.
- (3) $\Delta x_i = \underline{\hspace{2cm}} =$
- (4) It is most common to choose the subintervals to all have the same width, so
- (5) x_i^* denotes a chosen number in the interval $[x_{i-1}, x_i]$.
- (6) $\sum_{i=1}^n f(x_i^*)\Delta x_i$ is a _____
- (7) If $x_i^* = x_{i-1}$, then the Riemann sum is called the _____ Riemann sum.
- (8) If $x_i^* = x_i$, then the Riemann sum is called the _____ Riemann sum.
- (9) If $x_i^* = \frac{x_i - x_{i-1}}{2}$, then the Riemann sum is called the _____ Riemann sum.
- (10) If x_i^* is where the maximum occurs on $[x_{i-1}, x_i]$, then the Riemann sum is called the _____ Riemann sum.
- (11) If x_i^* is where the minimum occurs on $[x_{i-1}, x_i]$, then the Riemann sum is called the _____ Riemann sum.

Graphs

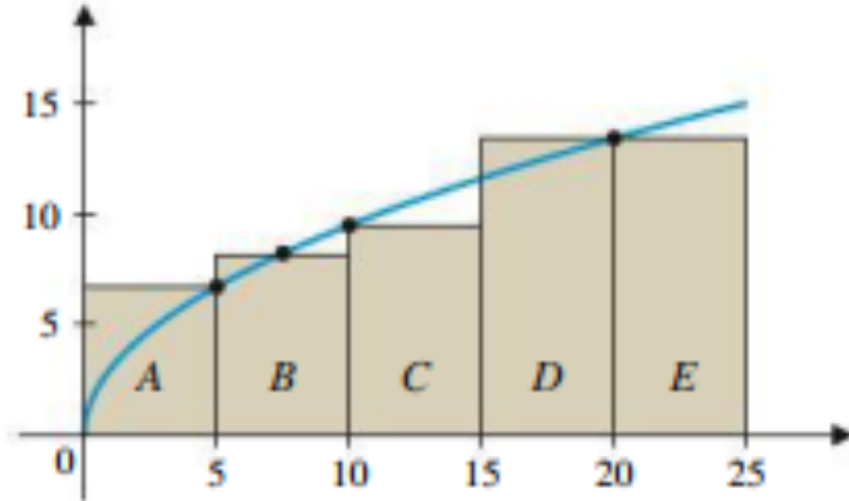
Example 5.4.1. *Left rule on $[0, 10]$ with 5 equal subintervals.*



Example 5.4.2. *Right rule on $[0, 10]$ with 5 equal subintervals.*



Example 5.4.3. Which rectangles below are left rectangles and which are right rectangles? Would the left Riemann sum be an overestimate or underestimate of the area under the curve? What about the right Riemann sum?



Integrals

(1) The **definite integral of f from a to b** is defined by

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i^*) \Delta x_i$$

- (a) f is called the _____
- (b) a and b are the _____ with b being the _____ while a is the _____
- (c) $f(x)$ is the _____
- (d) the process of finding the integral is _____

Properties

(1) If _____ then $\int_a^b f(x) dx$ is the exact area between the curve and the x axis over the interval $[a, b]$.

(2) If _____ then $\int_a^b f(x) dx$ is -1 times the exact area between the curve and the x axis over the interval $[a, b]$.

(3) $\int_a^b f(x) dx =$

(4) $\int_a^a f(x) dx =$

(5) $\int_a^b c dx =$

(6) $\int_a^b [f(x) + g(x)] dx =$

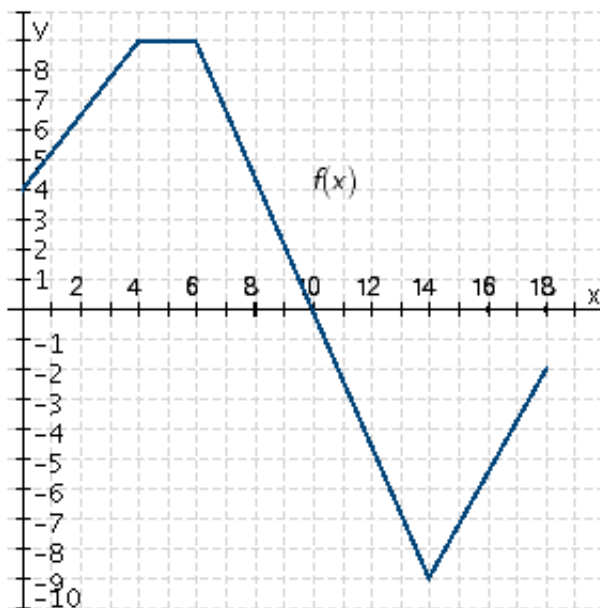
(7) $\int_a^b cf(x) dx =$

(8) $\int_a^b [f(x) - g(x)] dx =$

(9) $\int_a^c f(x) dx =$

Examples

Example 5.4.4. Use the graph to find the following integrals.



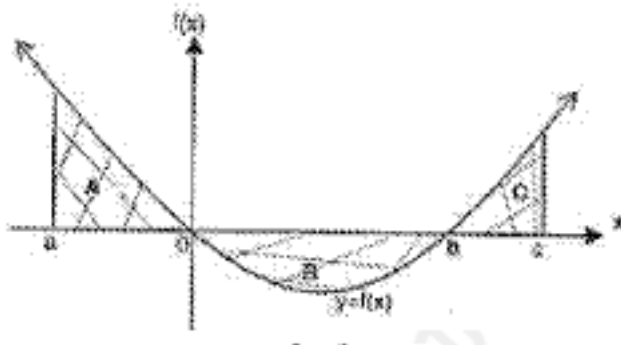
$$(1) \int_0^4 f(x) dx$$

$$(2) \int_{14}^{16} f(x) dx$$

$$(3) \int_6^{14} f(x) dx$$

$$(4) \int_6^4 f(x) dx$$

Example 5.4.5. Select ALL correct choices for the following graph with Area $A = 6$, Area $B = 15$, Area $C = 3$.



- (1) none of these
- (2) $\int_b^c f(x) dx = 3$
- (3) $\int_a^c f(x) dx = -6$
- (4) $\int_b^0 \frac{f(x) dx}{5} = -3$
- (5) $\int_b^a f(x) dx = 9$

Example 5.4.6. Given $\int_3^5 f(x) dx = 5$ and $\int_1^5 f(x) dx = 7$ find $\int_1^3 f(x) dx$.