## 5.1. Antiderivatives and Indefinite Integrals

(1) A function $F$ is called an ________________________ of $f$ on an interval $I$ of $F'(x) = f(x)$ for all $x$ in $I$.

(2) Theorem: If $F$ is an antiderivative of $f$ on an interval $I$ and $C$ is any constant, then $F(x) + C$ also defines an antiderivative of $f$ on $I$.

(3) If $F$ is an antiderivative of $f$, then we describe the antiderivative of a function in the most general terms by using the notation ________________________ to represent all possible antiderivatives of $f$.

(4) Notation: If $F(x)$ is an antiderivative of $f(x)$ then we write

## Rules for the Most General Antiderivative of $f$

(1) $\int k\,dx =$ (where $k$ is a constant)

(2) $\int f(x) \pm g(x)\,dx =$

(3) $\int kg(x)\,dx =$ (where $k$ is a constant)

(4) $\int x^n\,dx =$ (for $n \neq -1$)

(5) $\int x^{-1}\,dx =$

(6) $\int e^x\,dx =$

## Examples

**Example 5.1.1.** *Evaluate* $\int -6\,dx$.

**Example 5.1.2.** *Evaluate* $\int \, dx$.

**Example 5.1.3.** *Evaluate* $\int -4x^7\,dx$.

**Example 5.1.4.** *Evaluate* $\int (3 + 2u^{-4} - \sqrt{u})\,du$.

**Example 5.1.5.** *Evaluate* $\displaystyle\int \frac{-4}{z}\,dz$.

**Example 5.1.6.** *Evaluate* $\displaystyle\int t + 12e^t\,dt$.

**Example 5.1.7.** *Find* $y$ *if* $\dfrac{dy}{dx} = -6x^{-2} + x^{-1}$.

**Example 5.1.8.** *Find* $y$ *so that* $y(1) = -4$ *and* $\dfrac{dy}{dx} = -6x^{-2} + x^{-1}$.

**Example 5.1.9.** *Find the equation of the curve that passes through* $(1, 3)$ *if the slope is given by*

$$\frac{dy}{dx} = 12x^2 - 12x$$

*for each* $x$.

**Example 5.1.10.** *Evaluate* $\displaystyle\int \frac{x^3 + 4x^2 - 3x}{x^3}\, dx$.

**Example 5.1.11.** *The marginal average cost of producing $x$ smart watches is given by*

$$\overline{C}'(x) = -\frac{5000}{x^2} \qquad \overline{C}(100) = 250$$

*where $\overline{C}(x)$ is the average cost in dollars. Find the average cost function and cost functions. What are the fixed costs?*