

## 4.6. OPTIMIZATION

### Main Steps

- Step 1. Read problem and express all information from the problem mathematically. Use variables to represent any quantity that changes. Numbers may be used for quantities that remain constant.
- Step 2. Find a function for the quantity to be optimized in terms of one variable.
- Step 3. Find the absolute extreme required using the techniques from section 4.5.
- Step 4. re-read the problem and answer the question.

### Optimization Examples

**Example 4.6.1.** *Find two numbers whose difference is 10 and whose product is a minimum.*

**Example 4.6.2.** *If the price-demand and cost functions for a product are  $p = 500 - x$  and  $C(x) = \frac{1}{2}x^2 + 200x + 55$ , respectively, how many units,  $x$ , will maximize the profit?*

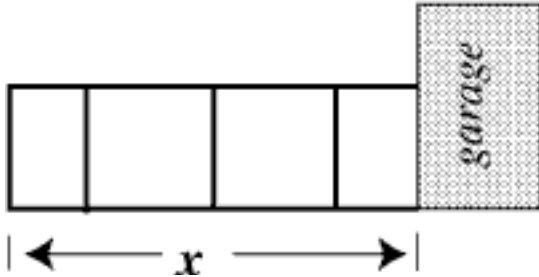
**Example 4.6.3.** *If a farmer plants 50 trees per acre, each tree will yield 150 bushels of peaches. For each additional tree he plants per acre, the yield of each tree will decrease by 3 bushels. If  $x$  is the number of ADDITIONAL trees planted per acre, find the total yield per acre in terms of  $x$ . Find the maximum yield.*

**Example 4.6.4.** *An airline sells 200 tickets per day at \$100 per ticket. Each \$5 price reduction will result in 20 more tickets sold per day. If  $x$  is the number of \$5 price reductions,*

(1) *find the total revenue,  $R$ , in terms of  $x$ .*

(2) *find the value for  $x$  that maximizes revenue.*

**Example 4.6.5.** A homeowner has 6400 feet of fencing material to use in enclosing three adjacent rectangular pens next to the garage. The side along the garage will not need fencing. (see figure) Find the total area of the enclosure  $A(x)$ , in terms of  $x$ .



**Example 4.6.6.** A homeowner has 6400 feet of fencing material to use in enclosing three adjacent rectangular pens next to the garage. The side along the garage will not need fencing. (see figure in example 4.6.5) Find the maximum total area of the enclosure  $A(x)$

**Example 4.6.7.** *A fence is to be built to enclose a rectangular area of 800 square feet. The fence along three sides is to be made of material that costs 6 per foot. The material for the fourth side costs 18 per foot. Find the dimensions of the rectangle that will allow for the most economical fence to be built.*

**Example 4.6.8.** *A candy box is made from a piece of cardboard that measures 37 by 20 inches. Squares of equal size will be cut out of each corner. The sides will then be folded up to form a rectangular box. What size square should be cut from each corner to obtain maximum volume?*