3.7. Elasticity of Demand

Definition 3.7.1. *The* **relative rate of change** *of a function* $f(x)$ *is* $\dfrac{f'(x)}{f(x)}$.

The **percentage rate of change** *is* $100 \times \dfrac{f'(x)}{f(x)}$.

Example 3.7.1. *Find the relative rate of change for* $f(x) = 9x - 5\ln x$ *at* $x = 7$

Theorem 3.7.1. *If price and demand are related by* $x = f(p)$*, then the elasticity of demand is given by*

$$E(p) = -\frac{pf'(p)}{f(p)}$$

Example 3.7.2. *The price* p *and the demand* x *for a product is related by the price-demand equation*

$$x + 500p = 10000$$

Find the elasticity of demand, $E(p)$.

$E(p)$	Demand	Interpretation	Revenue
$0 < E(p) < 1$	Inelastic	Demand is not sensitive to changes in price; that is, percentage change in price produces a smaller percentage change in demand.	A price increase will increase revenue.
$E(p) > 1$	Elastic	Demand is sensitive to changes in price; that is, a percentage change in price produces a larger percentage change in demand.	A price increase will decrease revenue.
$E(p) = 1$	Unit	A percentage change in price produces the same percentage change in demand.	

Example 3.7.3. *Use the price-demand equation to determine whether the demand is elastic, inelastic, or has unit elasticity for* $x = f(p) = 875 - p - 0.05p^2$ *at* $p = 50,\ 70,$ *and* 100*. Explain whether a price increase/decrease will increase/decrease revenue for each* p *value.*

Example 3.7.4. *The price-demand equation for an order of fries at a restaurant is*

$$x + 1000p = 2500$$

Currently, the price of an order of fries is 0.99*. If the price decreases by 10%, will revenue increase or decrease?*