3.4. The Chain Rule

Definition 3.4.1. *A function* f *is a* **composite** *of two functions* h *and* g *if*

$$f(x) = h \circ g = h[g(x)]$$

Theorem 3.4.1. *The derivative of a composite function,* $h \circ g$*, is*

$(h \circ g)'(x) =$

General Derivative Rules

(1) $\dfrac{d}{dx}[f(x)]^n = n[f(x)]^{n-1} f'(x)$

(2) $\dfrac{d}{dx} \ln[f(x)] = \dfrac{1}{f(x)} f'(x)$

(3) $\dfrac{d}{dx} e^{f(x)} = e^{f(x)} f'(x)$

(4) $\dfrac{d}{dx} \log_a[f(x)] = \left(\dfrac{1}{\ln a}\right) \dfrac{1}{f(x)} f'(x)$

(5) $\dfrac{d}{dx} a^{f(x)} = (\ln a) a^{f(x)} f'(x)$

Examples

Example 3.4.1. *Find the derivative and simplify* $f(x) = (2x + 3)^5$

Example 3.4.2. *If* $y = (5 - 2x^3 - x^6)^{-3}$ *find* y'

(1) $-3(5 - 2x^3 - x^6)^{-4}(-6x^2 - 6x^5)$
(2) $-3(5 - 2x^3 - x^6)^{-2}(-6x^2 - 6x^5)$
(3) $-3(-6x^2 - 6x^5)^{-4}$
(4) $-3(-6x^2 - 6x^5)^{-2}$
(5) *none of these*

Example 3.4.3. *Find the derivative of* $f(x) = \sqrt[4]{3x^2 - 4x + 5}$

Example 3.4.4. *If* $f(x) = e^{x^2+3x+1}$ *find* $f'(x)$

Example 3.4.5. *If* $f(x) = 5\ln(1 - x^3)$ *find* $f'(x)$

Example 3.4.6. *Find* y' *when* $y = \dfrac{e^{2x}}{3x - 2}$

Example 3.4.7. *Find* y' *when* $y = 8^{x^2-1}$

Example 3.4.8. *Find* y' *when* $y = \log_3(x - e^x)$

Chain Rule for Compositions of More Than Two "Simple" Functions

Example 3.4.9. *Find* $\dfrac{d}{dx}\sqrt[4]{\ln(2x+3)}$.

Example 3.4.10. *Find* $\dfrac{d}{dx}(1-e^{2t})^2$.

Example 3.4.11. *Find* $\dfrac{d}{dx}\ln(2x+3)^{3/2}$.

Example 3.4.12. *Find the equation of the line tangent to the graph of* $y = (x^2 - 3x + 2)^4$ *at* $x = 0$.

Example 3.4.13. *One of the value of x for which the graph of $f(x) = (x-1)(2-x)^3$ has a horizontal tangent line is*

(1) -2
(2) $-\frac{4}{5}$
(3) $\frac{5}{4}$
(4) $\frac{1}{2}$
(5) none of these

Example 3.4.14. *The total revenue from the sales of stereo speakers sold at \$p per stereo is given by $R(p) = 80p\sqrt{p+25} - 400, 20 \leq p \leq 100$. Find the instantaneous rate of change of $R(p)$ at $p = 75$.*