EXAMPLES USING THE GRAPHING STRATEGY.

SKETCH THE GRAPHS OF THE FOLLOWING FUNCTIONS. YOU MUST USE THE GRAPHING STRATEGY THAT IS OUTLINED IN YOUR TEXT ON PAGE 281. YOU MUST SHOW ALL OF THE STEPS MENTIONED IN THE GRAPHING STRATEGY.

1. Assume that $f(x)$ is continuous on $(-\infty, \infty)$. Use this information to sketch a graph of $f(x)$. (3 pt.)

$f(0) = 0,\ f(2) = -16,\ f(3) = -27;$

$f'(0) = 0,\ f'(3) = 0;$

$f'(x) < 0 \ on\ (-\infty, 0) \cup (0, 3);$

$f'(x) > 0 \ on\ (3, \infty);$

$f''(0) = 0,\ f''(2) = 0;$

$f''(x) < 0 \ on\ (0, 2);$

$f''(x) > 0 \ on\ (-\infty, 0) \cup (2, \infty)$

$f'(x)$ – – – – – – – – – – – – 0 – – – – – – – – – – – – – 0 + + + + + + + + + + + + + + +

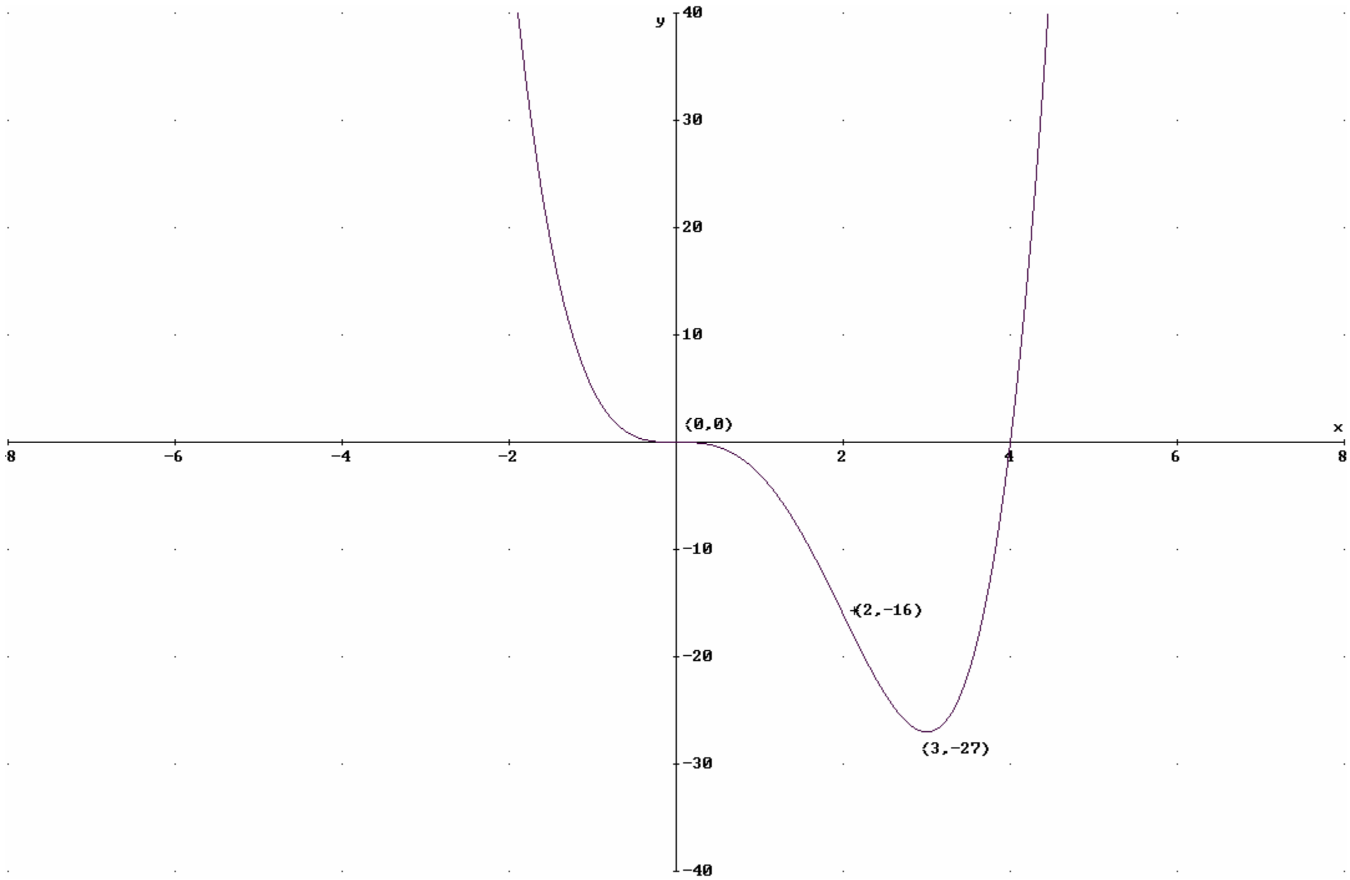
x

$f(x)$ DECR. 0 DECR. 3 INCR.

$f''(x)$ + + + + + + + + + + + 0 – – – – – – – – – – 0 + + + + + + + + + + + + + + + + + +

x

$f(x)$ CU 0 CD 2 CU



2. Sketch the graph of $f(x) = x^3 + 3x^2 - 9x - 11$.
(Don't worry about finding the **x-intercepts** for this function.) (3 pt.)

$Domain: (-\infty, \infty)$ No Asymptotes y-intercept: (0,–11) x-intercepts: $(-1,0), (-1+2\sqrt{3},0), (-1-2\sqrt{3},0)$

$f'(x) = 3x^2 + 6x - 9 = 3(x-1)(x+3) \Rightarrow f'(x)$ *exists* $\forall\, x \in Domain$

$f'(x) = 0 \Rightarrow x = 1$ *or* $x = -3 \Rightarrow$ *Critical values at* $x = 1$ *or* $x = -3$

$f'(x)$ + + + + + + + + + 0 - - - - - - - - - - - - - 0 + + + + + + + + + +

x

$f(x)$ INCR -3 DECR 1 INCR

Local Max. @ (-3, 16) Local Min. @ (1, -16)

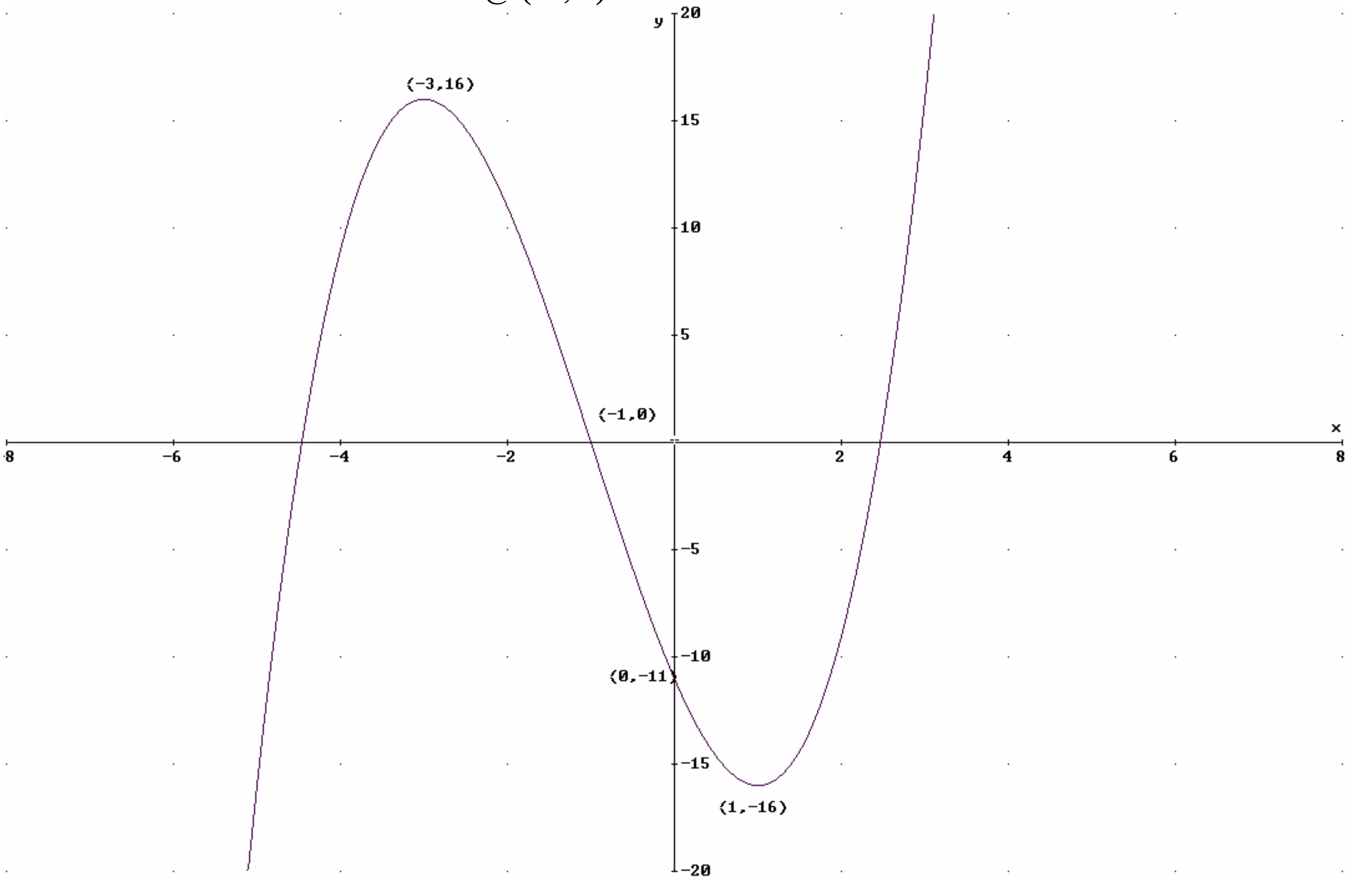
$f''(x) = 6x + 6 \Rightarrow f''(x)$ *exists* $\forall\, x \in Domain$

$f''(x) = 0 \Rightarrow x = -1$

$f''(x)$ - 0 + + + + + + + + + + + + + + +

$f(x)$ CD -1 CU → x

Inflection Point
@ (–1, 0)



3. Sketch the graph of $f(x) = \dfrac{5x^2}{x^2 - 1}$. (4 pt.)

Domain: $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$ y-intercept: (0, 0) x-intercepts: (0, 0) V.A.: x = ± 1 H.A.: y = 5

$$f'(x) = \frac{-10x}{\left(x^2 - 1\right)^2} \quad \Rightarrow \quad f'(x) \textit{ exists } \forall\ x \in \textit{Domain}$$

$$f'(x) = 0 \quad \Rightarrow \quad -10x = 0 \quad \Rightarrow \quad x = 0 \textit{ is only critical pt.}$$

$f'(x)$ + + + + + + + + N|D + + + + + 0 - - - - - - N|D - - - - - - - - - - -

$f(x)$ INCR -1 (o) INCR 0 DECR 1 (o) DECR → x

Local Max.
@ (0, 0)

$$f''(x) = \frac{30x^2 + 10}{\left(x^2 - 1\right)^3} \quad \Rightarrow \quad f''(x) \text{ exists } \forall\ x \in Domain$$

$$f''(x) = 0 \quad \Rightarrow \quad 30x^2 + 10 = 0 \quad \Rightarrow \quad \text{No real solutions} \quad \Rightarrow \quad \text{No Inflection pts.}$$

$f''(x)$	+ + + + + + + +	ND	- - - - - - - - - - - - - -	ND	+ + + + + + + + +	
		o		o		x
$f(x)$	CU	-1	CD	1	CU