8.2. Partial Derivatives

Definition 8.2.1. *Let $z = f(x,y)$ be a function of two variables. We define the* **partial derivative of f with respect to x** *as*

and the **partial derivative of f with respect to y** *as*

Remark 8.2.1. *Some common notations are*

Example 8.2.1. *Find $f_x(x,y)$ and $\frac{\partial z}{\partial y}$ if $z = f(x,y) = 5x^3y - 2xy^2 + 7x - 4y + 6$.*

Example 8.2.2. *Find $f_x(x,y)$ and $f_y(x,y)$ if $z = f(x,y) = xe^y + y\ln x + \frac{x}{y}$.*

Example 8.2.3. *Find* $f_x(x,y)$ *and* $f_y(x,y)$ *if* $z = f(x,y) = e^{2x^2 - xy + y^2}$.

Example 8.2.4. *Find* $f_x(x,y)$ *and* $f_y(x,y)$ *if* $z = f(x,y) = x^2 \ln(y^3 + xy)$.

Example 8.2.5. *Find* $R_p(p,q)$ *and* $R_q(p,q)$ *if* $R(p,q) = 12p - 4q + 4pq - p^4 + q^3$.

Second-order partial derivatives

Definition 8.2.2. *Let* $z = f(x, y)$ *be a function of two variables. We define the* **second order partial derivative of** f *are*

(1)

(2)

(3)

(4)

Example 8.2.6. *Find* $f_{xx}(x, y)$, $\frac{\partial^2 z}{\partial y^2}$, $f_{xy}(x, y)$, *and* $f_{yx}(x, y)$ *if* $z = f(x, y) = 5x^3y - 2xy^2 + 7x - 4y + 6$.

Example 8.2.7. *Find* $f_{yx}(x,y)$ *if* $z = f(x,y) = xe^y + y\ln x + \frac{x}{y}$.

Example 8.2.8. *Find* $f_{xx}(x,y)$ *if* $z = f(x,y) = e^{2x^2 - xy + y^2}$.

Example 8.2.9. *Find* $f_{xy}(x,y)$ *if* $z = f(x,y) = x^2\ln(y^3 + xy)$.

Example 8.2.10. *Find* $R_{qp}(p,q)$ *and* $R_{qq}(p,q)$ *if* $R(p,q) = 12p - 4q + 4pq - p^4 + q^3$.

Applications

Example 8.2.11. *A company manufactures two types of calculators,* A *and* B*. The weekly price-demand equations and cost equations are*

$$p = 15 - 2x + y$$

$$q = 20 + x - 2y$$

$$C(x,y) = 20 - 2x + y$$

where p *is the unit price of* A*,* q *is the unit price of* B*,* x *is the weekly demand for* A*,* y *is the weekly demand for* B*, and* $C(x,y)$ *is the cost function.*

(1) Find the profit function $P(x,y)$ *(in thousands of dollars).*

(2) Find $P_x(2,4)$

Remark 8.2.2. *In the above example, the solution to part (2) tells us that a production*

level of $x = 2$ *and* $y = 4$*, a unit increase in* ______ *while holding* ______ *fixed at*

______ *will increase/decrease the total profit be approximately* ______ *thousands of dollars.*

Example 8.2.12. *Let* $C(x, y)$ *be the cost function of* x *units of* A *and* y *units of* B*. In marginal analysis, what is the mathematical interpretation of* $C_y(7, 12) = -\$40$*?*

(1) *At a production level of* $x = 7$ *and* $y = 12$*, a unit increase in* x *while holding* y *fixed at 12 will decrease the total cost be approximately* \$40*.*

(2) *At a production level of* $x = 7$ *and* $y = 12$*, a unit increase in* y *while holding* x *fixed at 7 will decrease the marginal cost be approximately* \$40*.*

(3) *At a production level of* $x = 7$ *and* $y = 12$*, a unit decrease in* y *while holding* x *fixed at 7 will decrease the marginal cost be approximately* \$30*.*

(4) *At a production level of* $x = 7$ *and* $y = 12$*, a unit increase in* y *while holding* x *fixed at 7 will decrease the total cost be approximately* \$40*.*

(5) *At a production level of* $x = 7$ *and* $y = 12$*, a unit decrease in* y *while holding* x *fixed at 12 will decrease the total cost be approximately* \$40*.*

Homework: 8.2 p. 506 # 3, 7, 9, 13, 17, 21, 23, 25, 29, 31, 37, 47, 49, 63, 67 work e-grade practice at least 2 times.