4.3. Second Derivative and Graphs

Given $y = f(x)$, the derivative of the derivative is the ____________________________________.

Notation 4.3.1. $f''(x) = f^{(2)}(x) = y'' = \dfrac{d^2y}{dx^2} = D^2 f(x)$

The n-th derivative: $f^{(n)}(x) = \dfrac{d^ny}{dx^n} = D^n f(x)$

Example 4.3.1. *Find the first and second derivatives of the function.*

$f(x) = x^3 - 4x^2 + 3x - 10$

Applications

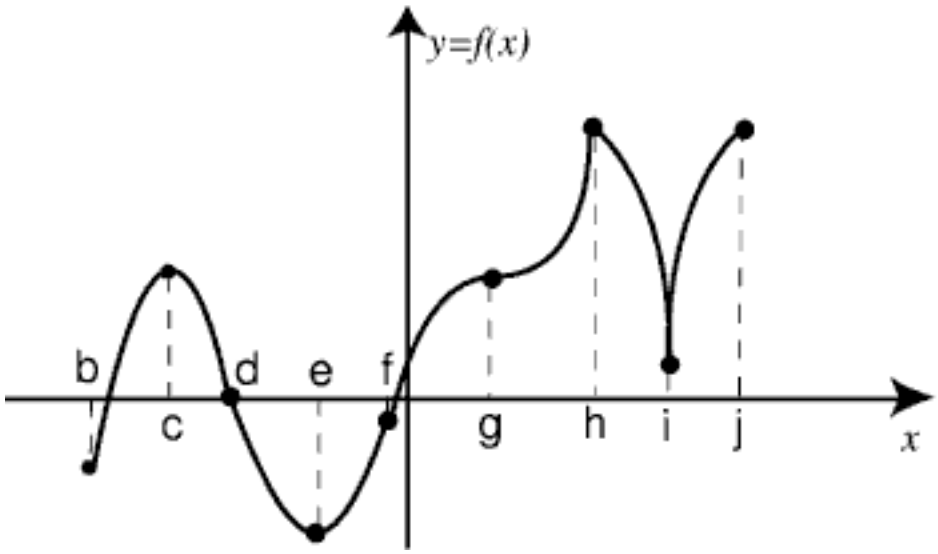
(1) Given the graph of $y = f(x)$
  (a) $f'(x)$ provides the slope of the the line tangent to $y = f(x)$ at x
  (b) $f''(x)$ provides the *rate of change* of the slope of the the line tangent to $y = f(x)$ at x.
  (c) thus $f''(x)$ tells us if the FIRST DERIVATIVE, $f'(x)$, is increasing or decreasing
  (d) so $f''(x)$ tells us if the tangent line is getting steeper or flatter.
  (e) and so $f''(x)$ tells us if the ORIGINAL FUNCTION, $f(x)$, is concave up or concave down.

(2) If $f(t)$ give the position of a particle at time, t, then
  (a) $f'(t)$ will provide the (instantaneous) __________________________ at time t and
  (b) $f''(t)$ will provide the (instantaneous) ____________________________________ at time t.
  (c) $f'''(t)$ will provide the ________________ at time t.

Concavity

Theorem 4.3.1.
(1) If $f''(x) > 0$ for all x in an interval I, then f is concave up on I.
(2) If $f''(x) < 0$ for all x in an interval I, then f is concave down on I.
(3) If f changes concavity at $x = c$ and f is defined at $x = c$, then we say $(c, f(c))$ is a **inflection point**. *To find inflection points we find where the second derivative changes signs (and is in the domain of the original function).*

Example 4.3.2. *The graph given is the graph of $y = f(x)$*



(1) *Find the intervals where the function is concave up and where concave down.*

(2) *Find the intervals where $f''(x) > 0$ and where $f''(x) < 0$*

(3) *Find the intervals where $f(x)$ is increasing and where $f(x)$ is decreasing*

(4) *Find the intervals where $f'(x)$ is increasing and where $f'(x)$ is decreasing*

(5) *Find where the inflection points occur*

(6) *Find the local extrema of $f(x)$*

(7) *Find the local extrema of $f'(x)$*

Theorem 4.3.2 (Second Derivative Test)**.** *Suppose $y = f(x)$ is such that $f'(c) = 0$ (and f is twice differentiable around c).*

(1) If $f''(c) > 0$ then ____________________________________

(2) If $f''(c) < 0$ then ____________________________________

(3) If $f''(c) = 0$ then ____________________________________

Example 4.3.3. *Find the local extrema of the function.*

$f(x) = x^3 - 4x^2 + 3x - 10$

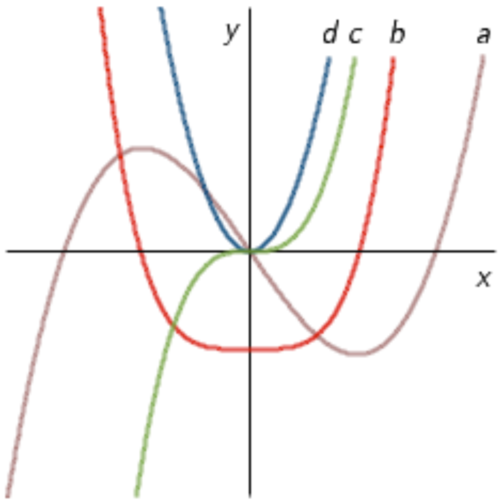
Example 4.3.4. *Find the local extrema of $f(x) = x^3 - 6x^2 - 15x + 10$*

Example 4.3.5. *Find the local extrema of $f(x) = -x + \frac{49}{x}$*

Putting it all together

Example 4.3.6. *Sketch a graph of* $f(x) = x^5 - 2x^3 + x$ *showing increasing, decreasing, local extrema, and concavity.*

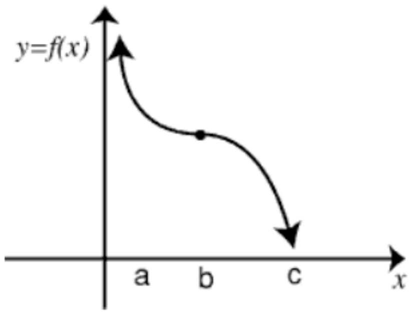
Example 4.3.7. *Given the graph below, identify f, f', f'', and f'''.*



Example 4.3.8. *Select ALL the correct choices for $f(x) = \frac{2}{3}x^3 - \frac{1}{2}x^2 - 3x + 4$*

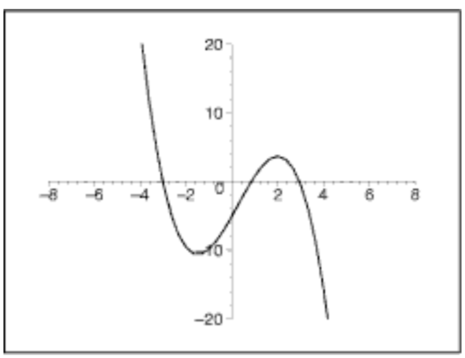
(1) *the graph of $f(x)$ has an inflection point at $x = \frac{1}{4}$*
(2) *the graph of $f(x)$ is concave downward on $(-\infty, \frac{1}{4})$*
(3) *the graph of $f(x)$ is concave downward on $(\frac{1}{4}, \infty)$*
(4) *the graph of $f(x)$ is increasing on $(-1, \frac{3}{2})$*
(5) *the graph of $f(x)$ is decreasing on $(-\infty, -1) \cup (\frac{3}{2}, \infty)$*
(6) *the graph of $f(x)$ has a local minimum at $x = \frac{3}{2}$*

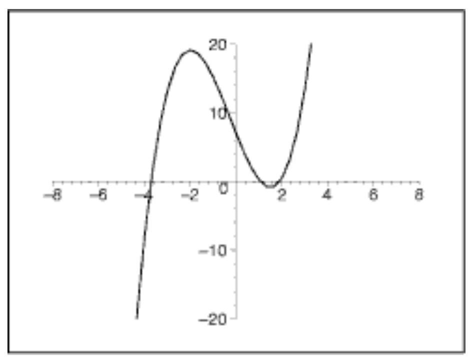
Example 4.3.9. *The graph given is the graph of $y = f(x)$. Choose the correct statement for the graph.*

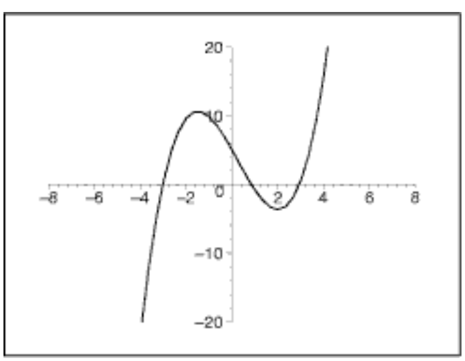


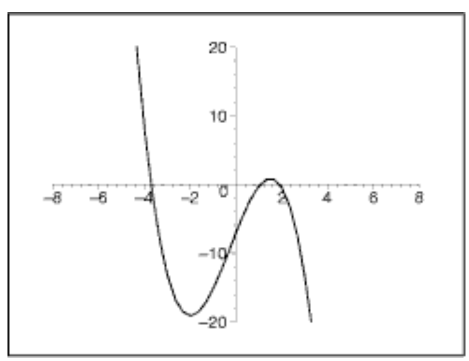
(1) $f'(x) > 0$ *on* (a, c); $f''(x) < 0$ *on* (a, b) *and* $f''(x) > 0$ *on* (b, c)
(2) $f'(x) > 0$ *on* (a, c); $f''(x) > 0$ *on* (a, b) *and* $f''(x) < 0$ *on* (b, c)
(3) $f'(x) < 0$ *on* (a, c); $f''(x) < 0$ *on* (a, b) *and* $f''(x) > 0$ *on* (b, c)
(4) $f'(x) < 0$ *on* (a, c); $f''(x) > 0$ *on* (a, b) *and* $f''(x) < 0$ *on* (b, c)

Example 4.3.10. *If* $f'(x) = 6 - x - 2x^2$ *and* $f''(x) = -1 - 4x$*, which of the following best represents the graph of* $f(x)$

(a)


(b)


(c)


(d)


Homework: 4.3 p. 270 # 1-8, 11, 15, 25, 31, 35, 55, 59, 85 work e-grade practice at least 2 times.