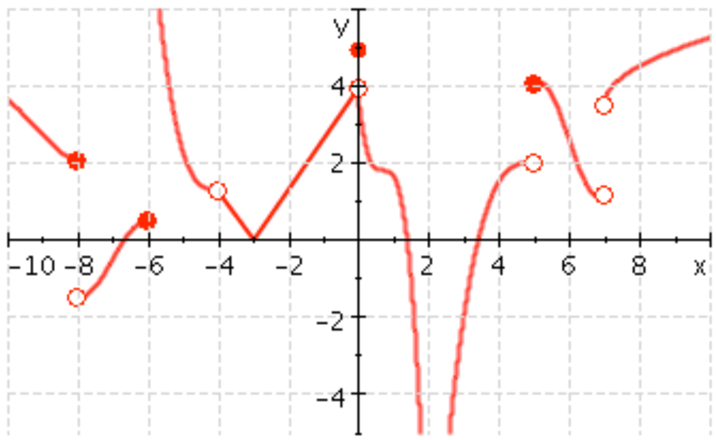
Definition 3.2.1 (Intuitive Definition)**.** *The* **limit of** $f(x)$**, as** x **approaches** a**, equals** L *means that as* x *gets arbitrarily close to the value* a *(but not actually equal to* a*), the value of* $f(x)$ *gets close to the value* L*. This is also written*

$$\lim_{x \to a} f(x) = L$$

Example 3.2.1. *Given the graph of* $y = f(x)$ *below*



(1) *Find* $\lim_{x \to a} f(x)$ *for* $a = -2, 0, 2, 5, -\infty$.

(2) *Find* $f(a)$ *for* $a = -2, 0, 2, 5$.

Left and Right Limits

Definition 3.2.2. *The* **limit of** $f(x)$**, as** x **approaches** a **from the left, equals** L *means that as* x *gets arbitrarily close to the value* a *AND* $x < a$*, the value of* $f(x)$ *gets close to the value* L*. This is also written*

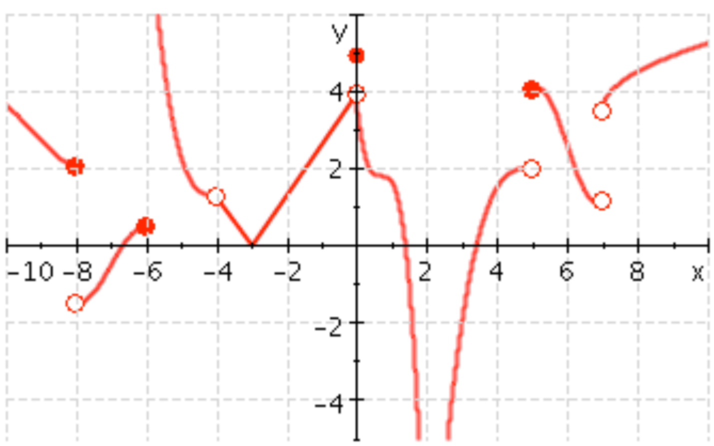
$$\lim_{x \to a^-} f(x)$$

Definition 3.2.3. *The* **limit of** $f(x)$**, as** x **approaches** a **from the right, equals** L *means that as* x *gets arbitrarily close to the value* a *AND* $x > a$*, the value of* $f(x)$ *gets close to the value* L*. This is also written*

$$\lim_{x \to a^+} f(x)$$

Theorem 3.2.1. $\lim_{x \to a} f(x) = L$ *if and only if*

Example 3.2.2. *Given the graph of* $y = f(x)$ *below*



(1) *Find* $\lim_{x \to a^-} f(x)$ *for* $a = -6, -4, -2, 5, 7$.

(2) *Find* $\lim_{x \to a^+} f(x)$ *for* $a = -6, -4, -2, 5, 7$.

(3) *Find* $f(a)$ *for* $a = -6, -4, -2, 5, 7$.

Example 3.2.3. *Consider the function* $f(x) = \frac{\sin 2x}{5x}$ *and find* $\lim_{x\to 0} f(x)$.

x	(sin(2x))/(5x)	x	(sin(2x))/(5x)
0.5	0.336588394	-0.5	0.336588394
0.1	0.397338662	-0.1	0.397338662
0.001	0.399999733	-0.001	0.399999733
0.0001	0.399999997	-0.0001	0.399999997
0.00001	0.4	-0.00001	0.4

Example 3.2.4. *Evaluate* $\lim\limits_{x\to -1}(3x+5)$

Example 3.2.5. *Evaluate* $\lim\limits_{x\to 2} x^5$

Example 3.2.6. *Evaluate* $\lim\limits_{x\to 8} -5$

Example 3.2.7. *Evaluate* $\lim\limits_{x\to 0^+} \dfrac{1}{x}$

Theorem 3.2.2 ("Common Sense" Limit Laws)**.** *If c is a constant and all limits involved exist (are real numbers), then*

(1) $\lim_{x\to a}[f(x)+g(x)] =$

(2) $\lim_{x\to a}[f(x)-g(x)] =$

(3) $\lim_{x\to a}[cf(x)] =$

(4) $\lim_{x\to a}[f(x)g(x)] =$

(5) $\lim_{x\to a}[f(x)/g(x)] =$

(6) $\lim_{x\to a}[f(x)]^n =$

(7) If f is a function that you know from previous experience is "continuous" at a, $\lim_{x\to a} f(x) =$

Example 3.2.8. *Find* $\lim_{x\to -1} 5x(x^2+3)$

Example 3.2.9. *Find* $\lim_{x\to 0} \frac{x^2-9}{x^2-x-6}$

Example 3.2.10. *Find* $\lim_{x\to -1/4} \frac{16x^2+1}{2-8x}$

Example 3.2.11. *Find* $\lim_{x \to 5} \sqrt[4]{3(47 - 4x)}$

Example 3.2.12. *Find* $\lim_{x \to -2} \frac{g(x) - 2f(x)}{3g(x)}$, *if* $\lim_{x \to -2} f(x) = 4$ *and* $\lim_{x \to -2} g(x) = -1$

More thought provoking limit law

Theorem 3.2.3. *If* $f(x) = g(x)$ *for all* x *in an open interval containing* x*, then* $\lim_{x \to a} f(x) = \lim_{x \to a} g(x)$

Example 3.2.13. *Find* $\lim_{x \to 3} \frac{x^2 - 9}{x^2 - x - 6}$

Example 3.2.14. *Find* $\lim_{x \to 4} \frac{x - 4}{x^2 - 2x - 8}$

Example 3.2.15. *Evaluate* $\lim_{t\to 0}\left[\frac{1}{t}-\frac{2}{t(t+2)}\right]$

Example 3.2.16. *Find* $\lim_{x\to 2}\left[\frac{2x-1}{x^2-x-2}-\frac{1}{x-2}\right]$

(1) the limit does not exist
(2) $1/3$
(3) 2
(4) none of these
(5) 3

Example 3.2.17. *Find* $\lim_{h\to 0}\frac{[3(x+h)^2-(x+h)]-[3x^2-x]}{h}$

Example 3.2.18. *Find* $\lim_{x\to 2^+}\frac{|2-x|}{2-x}$

Example 3.2.19. *Find* $\lim_{x\to 2^-}\frac{|2-x|}{2-x}$

Example 3.2.20. *Find* $\lim_{x\to 2}\frac{|2-x|}{2-x}$

Example 3.2.21. *f is given by*

$$f(x) = \begin{cases} -1 & \text{if } x < -2 \\ 2x+3 & \text{if } -2 \le x \le 1 \\ x^2 & \text{if } x > 1 \end{cases}$$

(1) *Evaluate* $\lim_{x\to 0} f(x)$

(2) *Evaluate* $\lim_{x\to 1^-} f(x)$

(3) *Evaluate* $\lim_{x\to 1^+} f(x)$

(4) *Evaluate* $\lim_{x\to 1} f(x)$

(5) *Evaluate* $f(1)$

(6) *Evaluate* $\lim_{x\to -2} f(x)$

Example 3.2.22. *Given*

$$f(x) = \begin{cases} 3-2mx & \text{if } x \le -2 \\ x-4m & \text{if } x > -2 \end{cases}$$

Find m such that $\lim_{x\to -2} f(x)$ exists.

Homework: 3.2 p. 158 # 1-17 eoo (every other odd), 23, 27, 31, 35-51 eoo, 57, 71, 75, work e-grade practice at least 2 times.