7.1. Area Between Curves

Recall the properties given in section 6.5:

(1) If $f(x) \geq 0$ on $[a, b]$ then $\int_a^b f(x)\,dx$ is the exact area between the curve and the x axis over the interval $[a, b]$.

(2) If $f(x) \leq 0$ on $[a, b]$ then $\int_a^b f(x)\,dx$ is -1 times the exact area between the curve and the x axis over the interval $[a, b]$.

(3) $\displaystyle\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$

So...

If $f(x) \geq 0$ on $[a, b]$ and $f(x) \leq 0$ on $[b, c]$, the area between $f(x)$ and the x-axis is

Theorem 7.1.1 (Area between two curves)**.** *If $f(x) \geq g(x)$ for all x in $[a, b]$, then the area between the graphs of f and g over the interval $[a, b]$ is*

$$\lim_{n\to\infty} \sum_{i=1}^{n} [f(x_i^*) - g(x_i^*)]\Delta x = \int_a^b f(x) - g(x)\,dx$$

In general, the area between the graphs of f and g over the interval $[a, b]$ is

Examples

Example 7.1.1. *Find the area bounded by the graphs* $y = e^{-x}$ *and* $y = 0$ *for* $-1 \leq x \leq 0$.

Example 7.1.2. *Find the area between the graphs* $y = -6x - 2$ *and* $y = 10$ *for* $-2 \leq x \leq 2$.

Example 7.1.3. *Find the area between the graphs* $y = -6x - 2$ *and* $y = 10$ *for* $-4 \leq x \leq 0$.

Example 7.1.4. *Find the area between the graphs* $y = -4x^2 - 8x + 5$ *and* $y = 3x^2 - 8x - 2$.

Example 7.1.5. $y = 1 + \sqrt{x}$ *and* $y = \dfrac{3 + x}{3}$

Homework: 7.1 p. 454 # 3, 11, 13, 15, 19, 23, 33, 39, 47, 67 work e-grade practice at least 2 times.