3.7. Marginal Analysis in Business and Economics

Definitions 3.7.1. *If x is the number of units of a product produced during some time interval, then*

- ______________________________ *is the instantaneous rate of change of the Cost relative to the production at a given production rate.*

  In other words, if Total Cost is $C(x)$, then the ______________________________ *is $C'(x)$.*

- ______________________________ *is the instantaneous rate of change of the Revenue relative to the production at a given production rate.*

  In other words, if Total Revenue is $R(x)$, then the ______________________________ *is $R'(x)$.*

- ______________________________ *is the instantaneous rate of change of the Profit relative to the production at a given production rate.*

  In other words, if Total Profit is $P(x) = R(x) - C(x)$, then the ______________________________ *is $P'(x) = R'(x) - C'(x)$.*

- *If $p = f(x)$ is the price-demand equation relating the price p to the demand x, then the revenue is given by $R =$*

Example 3.7.1. *If the profit for producing x items is given by $P(x) = -\frac{1}{4}x^2 + 15x - 5000$, find the marginal profit function y.*

Example 3.7.2. *If the total profit for producing x items is given by $P(x) = -\frac{1}{4}x^2 + 15x - 5000$, find the marginal profit at $x = 200$.*

Example 3.7.3. *The price p (in dollars) and the demand x for a product are related by the equation $x = 24 - 8p$. Find the revenue function, $R(x)$.*

Example 3.7.4. *For a particular product, the price-demand equation is $p = -\frac{5}{7}x + 1300$, where p is the price and x is the quantity, and the cost function is $C(x) = 4000 + 3x$. What is the profit function, $P(x)$?*

Example 3.7.5. *The price p (in dollars) and the demand x for a product are related by the equation $x = 24 - 8p$. Find the marginal revenue function, $R'(x)$.*

Marginal v.s. exact

Theorem 3.7.1. *The Marginal Cost of producing x items approximate the exact cost of producing the $(x+1)$-th item. In other words*

Similar statement may be made about profit and revenue.

Example 3.7.6. *The total cost (in dollars) of producing x units of a product is given by the function $C(x)$. Applying marginal analysis, which of the following would best be used to approximate the cost of producing the 18th unit?*

(1) $C'(18)$
(2) $C'(17)$
(3) $C'(18) - C'(17)$
(4) $C'(19) - C'(18)$

Example 3.7.7. *The total profit (in dollars) of producing x units of a product is given by the function $P(x)$. Which of the following statements best represents a correct interpretation of $P'(66) = -10$?*

(1) At a production level of 66 *units, a unit increase in productions will decrease total profit by approximately* \$10.
(2) At a production level of 66 *units, a unit increase in productions will decrease marginal profit by approximately* \$10.
(3) At a production level of 66 *units, the total profit is approximately* \$10.
(4) At a production level of 66 *units, a unit increase in productions will increase total profit by approximately* \$10.
(5) At a production level of 66 *units, a unit increase in productions will increase marginal profit by approximately* \$10.
(6) At a production level of 10 *units, the total profit by approximately* \$66.

Average

Definitions 3.7.2. *If x is the number of units produced in some time interval, then*

- *The* **Average Cost function**, $\overline{C}(x)$, *is the Cost function divided by x. In other words*
$$\overline{C}(x) = \frac{C(x)}{x}$$

- *The* **Marginal Average Cost function**, $\overline{C}'(x)$, *is the derivative of the Average Cost Function. In other words, it is*
$$\overline{C}'(x)$$

- *The* **Average Revenue function**, $\overline{R}(x)$, *is the Revenue function divided by x. In other words*
$$\overline{R}(x) = \frac{R(x)}{x}$$

- *The* **Marginal Average Revenue function**, $\overline{R}'(x)$, *is the derivative of the Average Revenue Function. In other words, it is*
$$\overline{R}'(x)$$

- *The* **Average Profit function**, $\overline{P}(x)$, *is the Profit function divided by x. In other words*
$$\overline{P}(x) = \frac{P(x)}{x}$$

- *The* **Marginal Average Profit function**, $\overline{P}'(x)$, *is the derivative of the Average Profit Function. In other words, it is*
$$\overline{P}'(x)$$

Example 3.7.8. *If the cost function is given by $C(x) = 300 - 100x + \frac{x^2}{20}$, find the average cost function, $\overline{C}(x)$.*

Example 3.7.9. *If the cost function is given by $C(x) = 300 - 100x + \frac{x^2}{20}$, find the marginal average cost function, $\overline{C}'(x)$.*

Example 3.7.10. *If the average revenue function for a product is given by $\overline{R}(x) = -\frac{x}{4} + 300 + \frac{1000}{x}$, find the revenue function, R.*

(1) $R(x) = -\frac{x^2}{4} + 300x + 1000$
(2) $R(x) = -\frac{1}{4} + 300x + \frac{1000}{x^2}$
(3) $R(x) = -\frac{x^2}{4} + 300x + \frac{1000}{x}$
(4) $R(x) = -\frac{1}{4} + \frac{300}{x} + \frac{1000}{x^2}$
(5) *none of these*

Homework: 3.7 p. 213 # 3, 9, 11, 13 work e-grade practice at least 2 times.