3.3. The Derivative

Definition 3.3.1. *Recall from 3.1: Given a function* $y = f(x)$*, a* **difference quotient** *is an expression of the form*

Example 3.3.1. *If* $f(x) = 3 - 2x^2$*, find* $\dfrac{f(2) - f(-5)}{2 - (-5)}$.

Example 3.3.2. *Given* $f(x) = 2 - 2x - x^2$*, find* $\dfrac{f(-1+h) - f(-1)}{h}$, $h \neq 0$.

Example 3.3.3. *Given* $f(x) = 2 - 2x - x^2$*, find* $\lim_{h \to 0} \dfrac{f(-1+h) - f(-1)}{h}$.

Definition of the Derivative

Definition 3.3.2. *The line ________________________ to a curve at a point is the line the "best approximates" the curve at that point.*

Definition 3.3.3. *The ____ of an object over at a given time, a is the limiting value of the average velocity over the time interval from t to a as t approaches a.*

Definition 3.3.4. *For the following we assume $f(x) = y$ is a function.*

(1) *The* **derivative of f with respect to** x *at* $x = a$ *is*

$$f'(a) = y'(a) = \frac{d}{dx}f(x)\Big|_{x=a} = \frac{dy}{dx}\Big|_{x=a} = Df(a) = D_x f(a) =$$

(2) *The* **derivative of f with respect to** x *is*

$$f'(x) = y' = \frac{d}{dx}f(x) = \frac{dy}{dx} = Df(x) = D_x f(x) =$$

(3) *The second derivative of f with resect to x is the derivative of f' with respect to x.*

Remark 3.3.1. *All of the following concepts are found using the derivative:*

(1) *the slope of a tangent line,*
(2) *velocity of a particle using the position,*
(3) *the acceleration of a particle using velocity,*
(4) *instantaneous rate of change of a quantity*
(5) *marginal cost using a cost function*
(6) *marginal revenue using a revenue function*

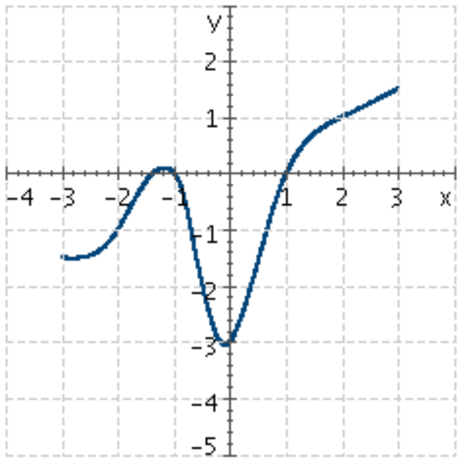
Example 3.3.4. *Given* $f(x) = 2 - 2x - x^2$, *find* $f'(x)$.

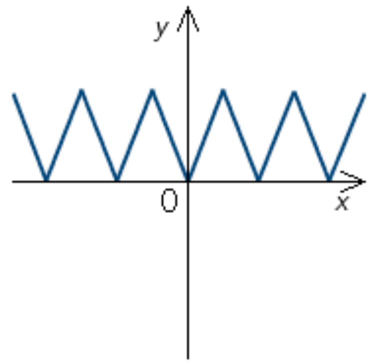
Example 3.3.5. *Given* $f(x) = 1/x$, *find* $f'(x)$.

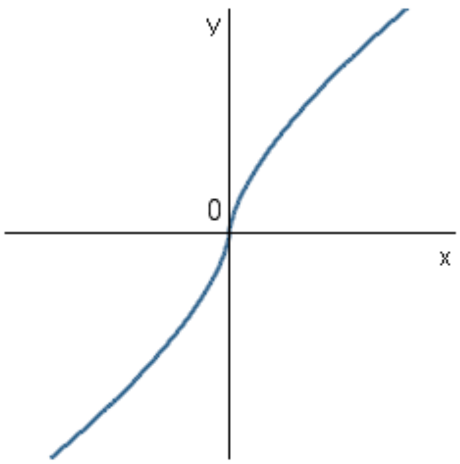
Example 3.3.6. *Use the following expression and the definition of the derivative to find* $f'(x)$*:*

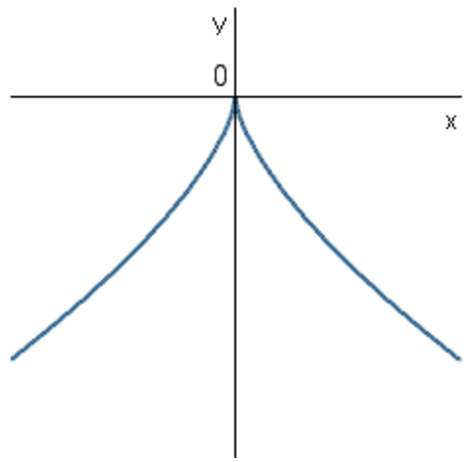
$$f(x+h) - f(x) = 4x^2h - 3xh + xh^2 + 2h^3 - h.$$

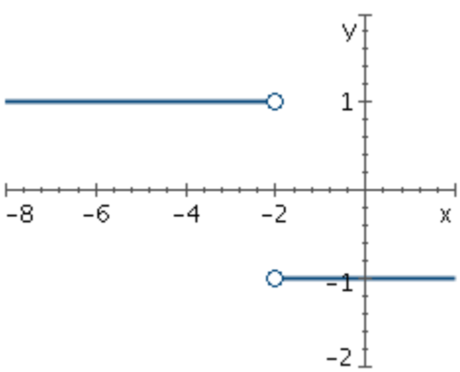
Example 3.3.7. *Discuss the differentiability*

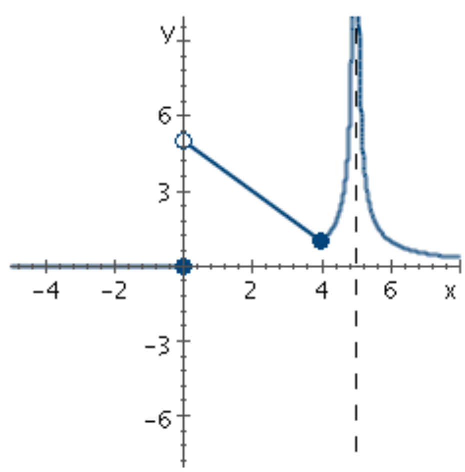












Example 3.3.8. *The total saes of a company (in millions of dollars) t months from now are given by* $S(t) = \sqrt{t+6}$. *Find* $S(10)$ *and* $S'(10)$, *and interpret. Use these results to estimate the total sales after 13 months and 14 months.*

Homework: 3.3 p. 171 # 5, 11, 13, 19, 27-37 odd, 53, 57, 61, 65, work e-grade practice at least 2 times.