Section 8.5: The Dot Product

If $\vec{v} = a_1 \vec{i} + b_1 \vec{j}$ and $\vec{w} = a_2 \vec{i} + b_2 \vec{j}$

Then $\vec{v} \cdot \vec{w} =$

Properties:

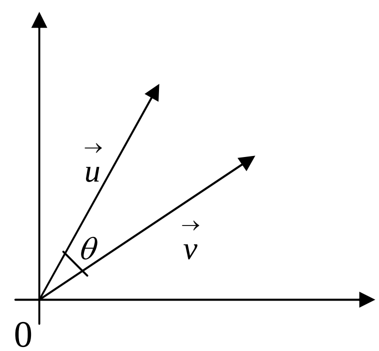
1) $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$
2) $\vec{v} \cdot \vec{v} = \| \vec{v} \|^2$ $\quad [\ \| \vec{v} \|^2 = a_1^{\ 2} + b_1^{\ 2}\ ]$
3) $\vec{0} \cdot \vec{v} = 0$

Ex: If $\vec{v} = 2\vec{i} + 5\vec{j}$ and $\vec{w} = 4\vec{i} - 3\vec{j}$
Find 1) $\vec{v} \cdot \vec{w}$, 2) $\vec{v} \cdot \vec{v}$, 3) $\vec{w} \cdot \vec{w}$

Theorem: Angle between vectors

If $\vec{u}$ and $\vec{v}$ are two nonzero vectors, the angle θ, $0 \le \theta \le \pi$ between $\vec{u}$ and $\vec{v}$ is determined by the formula

$$\cos\theta = \frac{\vec{u} \cdot \vec{v}}{\| \vec{u} \| \| \vec{v} \|}$$



Notes:

1) $\vec{v}$ and $\vec{u}$ are parallel if $\theta = 0$ or $\theta = \pi$ [i.e. $\vec{v} = \alpha \vec{u}$ one vector is scalar multiple of the other]
   a) $\vec{v}$ and $\vec{u}$ are in the same direction if $\theta = 0$
   b) $\vec{v}$ and $\vec{u}$ are in the opposite direction if $\theta = \pi$
2) $\vec{v}$ and $\vec{u}$ are orthogonal if $\theta = \frac{\pi}{2}$

Theorem: Two nonzero vectors are orthogonal if and only if

$$\vec{v} \cdot \vec{u} = 0$$

Ex: Find the angle between the give two vectors.

1) $\vec{v} = 3\,\vec{j}$, $\vec{w} = 2\,\vec{i} + 2\,\vec{j}$

2) $\vec{v} = 5\,\vec{i} - 2\,\vec{j}$, $\vec{w} = 2\,\vec{i} + 5\,\vec{j}$

3) $\vec{v} = \sqrt{3}\,\vec{i} - \sqrt{3}\,\vec{j}$, $\vec{w} = \sqrt{6}\,\vec{j}$

Ex: Determine if the given two vectors are parallel, orthogonal , or neither.

1) $\vec{v} = 2\,\vec{i} - \vec{j}$, $\vec{w} = 4\,\vec{i} - 2\,\vec{j}$

2) $\vec{v} = 3\,\vec{i} - 5\,\vec{j}$, $\vec{w} = -\frac{12}{7}\,\vec{i} + \frac{20}{7}\,\vec{j}$

3) $\vec{v} = 4\,\vec{i} - \vec{j}$, $\vec{w} = 2\,\vec{i} + 8\,\vec{j}$

4) $\vec{v} = 4\,\vec{i} - 3\,\vec{j}$, $\vec{w} = \vec{i} + 2\,\vec{j}$

5) $\vec{v} = 8\,\vec{i} - 4\,\vec{j}$, $\vec{w} = -6\,\vec{i} - 12\,\vec{j}$

6) $\vec{v} = \frac{1}{2}\,\vec{i} - 3\,\vec{j}$, $\vec{w} = -\vec{i} + 6\,\vec{j}$

Ex: Determine m such that the two vectors are orthogonal.

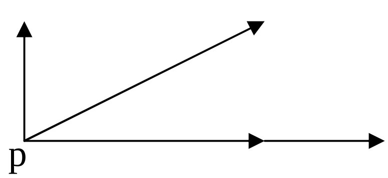
1) $\vec{v} = 4m\,\vec{i} + \vec{j}$, $\vec{u} = 9m\,\vec{i} - 25\,\vec{j}$

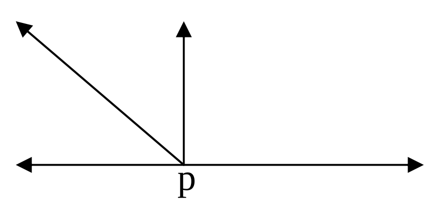
2) $\vec{v} = 3\,\vec{i} - 2\,\vec{j}$, $\vec{u} = 4\,\vec{i} + 5m\,\vec{j}$

3) $\vec{v} = (m-1)\,\vec{i} - 3\,\vec{j}$, $\vec{u} = \vec{i} + m\,\vec{j}$

Projection of a Vector onto another Vector

Note:





Theorem: If $\vec{v}$ and $\vec{w}$ are two nonzero vectors, then the vector projection of $\vec{v}$ onto $\vec{w}$ is

$$proj_{\vec{w}}\,\vec{v} = \frac{\vec{v}\cdot\vec{w}}{\|\vec{w}\|^2}\,\vec{w}$$

The decomposition of $\vec{v}$ into $\vec{v}_1$ and $\vec{v}_2$ where

$\vec{v}_1$ is parallel to $\vec{w}$

And $\vec{v}_2$ is orthogonal to $\vec{w}$ is

$$\vec{v}_1 = proj_{\vec{w}}\, \vec{v} = \frac{\vec{v} \cdot \vec{w}}{\|\vec{w}\|^2}\vec{w}$$

And $\vec{v}_2 = \vec{v} - \vec{v}_1$

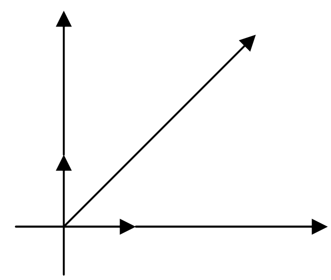
<u>Ex:</u> Given the two vectors

1) $\vec{v} = 2\vec{i} - 3\vec{j}$, $\vec{w} = \vec{i} - \vec{j}$

2) $\vec{v} = -\vec{i} + 2\vec{j}$, $\vec{w} = 3\vec{i} - \vec{j}$

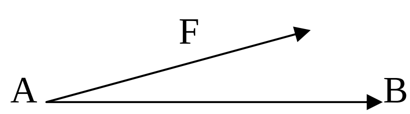
Find: a) The projection of $\vec{v}$ on $\vec{w}$; b) The projection of $\vec{v}$ orthogonal to $\vec{w}$

<u>Writing a Vector in terms of its magnitude and direction:</u>



<u>Work Done By Constant Force:</u>

$W = \vec{F} \cdot \vec{AB}$ **<u>unit:</u>** ft-pound



<u>Ex:</u>

1) Find the work done by the force of 3 pounds acting in the direction $2\vec{i} + \vec{j}$ in the moving an object 2 feet from $(0,0)$ to $(0,2)$.

2) Find the work done by the force of 1 pound acting in the direction $2\vec{i} + 2\vec{j}$ in the moving an object 5 feet from $(0,0)$ to $(3,4)$.