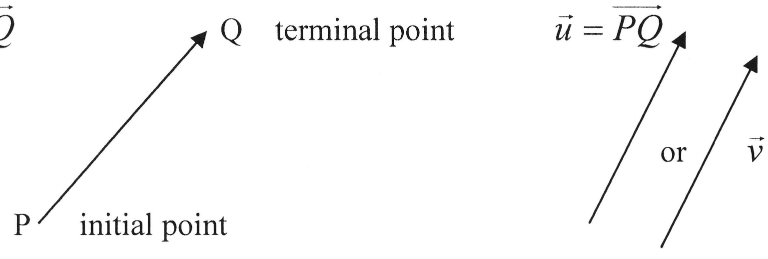
Section 8.4: Vectors

I) Area, Volume, Distance, Temperature … have magnitude only (scalar quantities)
II) Velocity, Force both have magnitude and direction represent by directed line segment which is called vector.

1) We write $\overrightarrow{PQ}$

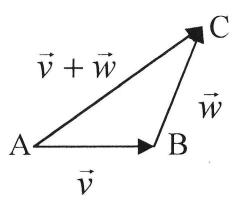


2)Length of $\overrightarrow{PQ}$ is $\left\|\overrightarrow{PQ}\right\|$ or $\|\vec{u}\|$

3) Vectors with the same magnitude and direction are equivalent $\vec{u} = \vec{v}$

4) Zero Vectors $\vec{0}$ (i.e. magnitude is zero)

5)



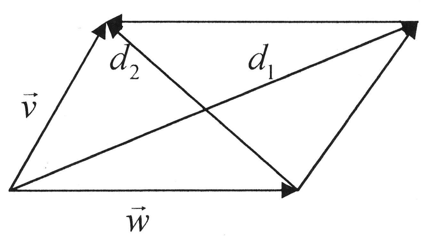
$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$ (adding)

6) Commutative and associative laws will apply.

7) $\vec{v} + \vec{0} = \vec{0} + \vec{v} = \vec{v}$

8) $\vec{v} + (-\vec{v}) = \vec{0}$

9)

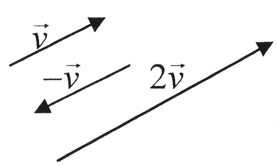


Find d_1 and d_2 ?

Multiplying vectors by numbers:

If $\alpha = real\ number \Rightarrow \alpha\vec{v}$ is a vector whose magnitude $\|\alpha\vec{v}\| = |\alpha|\|\vec{v}\|$

a) Direction same as $\vec{v}$ if $\alpha > 0$
b) Direction opposite to $\vec{v}$ if $\alpha < 0$
c) $\alpha\vec{v}$ scalar multiple of $\vec{v}$



Theorem: Properties of $\|\vec{v}\|$

$\alpha = scalar$

a) $\|\vec{v}\| \geq 0$
b) $\|\vec{v}\| = 0 \Leftrightarrow \vec{v} = 0$
c) $\|-\vec{v}\| = \|\vec{v}\|$
d) $\|\alpha\vec{v}\| = |\alpha|\|\vec{v}\|$

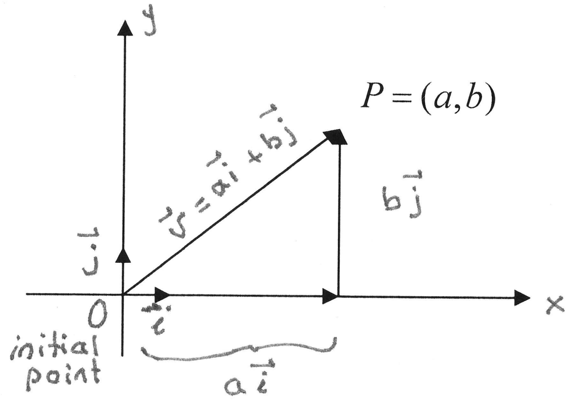
Note: A vector $\vec{u}$ for which $\|\vec{u}\| = 1$ is called a unit vector.

Representing vectors in the plane:

Two unit vectors;

One Parallel to x-axis called $\vec{i}$

One Parallel to y-axis called $\vec{j}$



$\vec{v} = a\vec{i} + b\vec{j}$

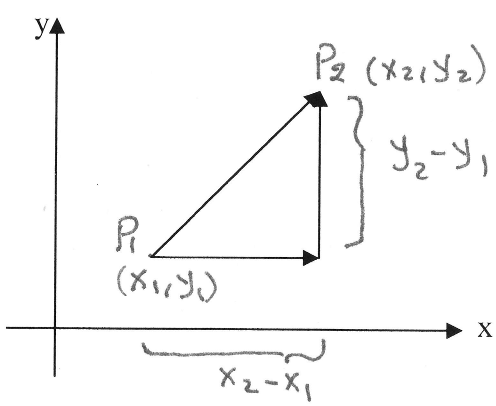
a and b are called components of the vector $\vec{v}$

a is in the direction $\vec{i}$

b is in the direction $\vec{j}$

Theorem: Suppose the $\vec{v}$ is a vector with the initial point $P_1 = (x_1, y_1)$ not necessarily the origin, and the terminal point $P_2 = (x_2, y_2)$. If $\vec{v} = \overrightarrow{P_1P_2}$ then $\vec{v}$ is equal to the position vector

$\vec{v} = (x_2 - x_1)\vec{i} + (y_2 - y_1)\vec{j}$



EX: If $P = (-3,2)$ and $Q = (6,5)$ find 1) $\overrightarrow{PQ}$, 2) $\overrightarrow{QP}$

Theorem:

If $\vec{v} = a_1 \vec{i} + b_1 \vec{j}$ and $\vec{w} = a_2 \vec{i} + b_2 \vec{j}$

Then $\vec{v} = \vec{w} \Leftrightarrow a_1 = a_2 \ and \ b_1 = b_2$

Notes:

1) $\|\vec{v}\| = \sqrt{a_1^2 + b_1^2}$

2) $\vec{v} + \vec{w} = (a_1 + a_2)\vec{i} + (b_1 + b_2)\vec{j}$

3) $\alpha\vec{v} = (\alpha a_1)\vec{i} + (\alpha b_1)\vec{j}$

EX: If $\vec{v} = 3\vec{i} - \vec{j}$ *and* $\vec{w} = -2\vec{i} + 3\vec{j}$

Find 1) $\vec{v} - \vec{w}$, 2) $\|2\vec{w} - \vec{v}\|$, 3) $\|2\vec{v} - 3\vec{w}\|$, 4) $\|2\vec{w}\| - \|\vec{v}\|$

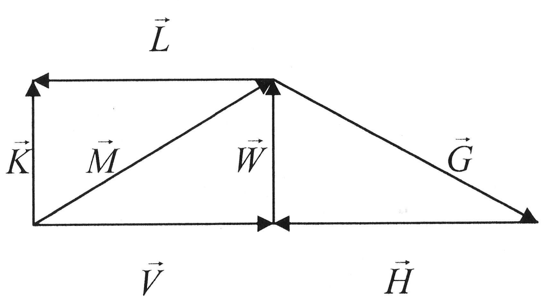
Theorem: Unit Vector in direction of $\vec{v}$

For nonzero vector $\vec{v}$, the vector $\vec{u} = \dfrac{\vec{v}}{\|\vec{v}\|}$ is the unit vector that has the same direction as $\vec{v}$

EX: Find the unit vector having the same direction as $\vec{v}$

1) $\vec{v} = 2\vec{i} - \vec{j}$, 2) $\vec{v} = -5\vec{i} + 12\vec{j}$

EX: Use the figure below to answer True or False



1) $\vec{V} + \vec{W} + \vec{L} = \vec{K}$

2) $\vec{H} + \vec{G} = \vec{M} - \vec{V}$