6.3: Double-Angle and Half- Angle formulas

<u>Double-Angle formulas:</u>

1) $\sin 2\theta =$

2) $\cos 2\theta =$

$\quad\quad =$

$\quad\quad =$

3) $\tan 2\theta =$

<u>**Ex:**</u> Given $\cot\theta = \dfrac{4}{3}$, $\pi < \theta < \dfrac{3\pi}{2}$. Find $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$

<u>**Ex:**</u> Evaluate

a) $\csc[2\cos^{-1}(-\dfrac{2}{3})]$,   b) $\tan[2\sin^{-1}(-\dfrac{4}{5})]$,   c) $\cos[2\tan^{-1}(-\sqrt{3})]$

<u>**Ex:**</u> a) If $\cos 2x = \dfrac{7}{9}$ find $\tan x$ for $\pi < x < \dfrac{3\pi}{2}$

b) If $\tan 2x = -\dfrac{4}{3}$ find $\sin x$ for $-\dfrac{\pi}{2} < x < 0$

c) If $\sin 2x = -\dfrac{3}{5}$, find $a)\tan x$, $b)\sec x$ for $-\pi < 2x < -\dfrac{\pi}{2}$

<u>**Ex:**</u> Find the exact value for the following.

1) $1 - 2\sin^2\dfrac{7\pi}{12}$, 2) $\cos^2\dfrac{\pi}{8} - \sin^2\dfrac{\pi}{8}$, 3) $\sin 165^o \cos 165^o$,

4) $2\cos^2\dfrac{5\pi}{12} - 1$, 5) $2\sin\dfrac{\pi}{12}\cos\dfrac{\pi}{12}$, 6) $\dfrac{2\tan\dfrac{7\pi}{8}}{1 - \tan^2\dfrac{7\pi}{8}}$

Notes:

     1) $\sin^2\theta =$

     2) $\cos^2\theta =$

     3) $\tan^2\theta =$

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Notes:

     1) $\sin^2\dfrac{\alpha}{2} =$

     2) $\cos^2\dfrac{\alpha}{2} =$

     3) $\tan^2\dfrac{\alpha}{2} =$

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Half-Angle formulas

     1) $\sin\dfrac{\alpha}{2} =$

     2) $\cos\dfrac{\alpha}{2} =$

     3)* $\tan\dfrac{\alpha}{2} =$

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Note: For $\tan\dfrac{\alpha}{2}$ use the following formulas:

$$\tan\frac{\alpha}{2} = \Bigg\langle$$

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Ex: 1) If $\sec\dfrac{\theta}{2} = -\dfrac{5}{3}$, *find* $\tan\dfrac{\theta}{2}$ *given* $-3\pi < \theta < -2\pi$

    2) If $\cot 2\theta = -\dfrac{8}{15}$, *find* $\sin 2\theta$ *given* $\dfrac{11\pi}{12} < \theta < \pi$

Ex: Find the exact value for the following.

1) $\sin(-\dfrac{\pi}{8})$, 2) $\sin(-\dfrac{29\pi}{12})$, 3) $\cos(\dfrac{19\pi}{8})$,

4) $\tan(\dfrac{21\pi}{8})$, 5) $\cos(-\dfrac{41\pi}{12})$, 6) $\sin(-\dfrac{37\pi}{12})$

Ex: Evaluate

1) $\tan[\dfrac{1}{2}\cos^{-1}(-\dfrac{1}{2})]$, 2) $\sin[\dfrac{1}{2}\tan^{-1}(-2\sqrt{2})]$

Ex: 1) Given $-2\pi < \theta < 0$. If $\sin\theta = \dfrac{4}{5}$ and $\tan\theta > 0$, find $\cos\dfrac{\theta}{2}$

2)) Given $0 < \theta < 2\pi$. If $\tan\theta = -\sqrt{8}$ and $\sec\theta > 0$, find $\sin\dfrac{\theta}{2}$

Ex: Write as a single Trig. Function value using half angle or double angle formulas.

1) $-\sqrt{\dfrac{1+\cos 200^o}{2}}$, 2) $\dfrac{1-\cos\dfrac{\pi}{7}}{\sin\dfrac{\pi}{7}}$, 3) $\dfrac{2\tan\dfrac{\theta}{2}}{1-\tan^2\dfrac{\theta}{2}}$, 4) $-\sqrt{\dfrac{1-\cos\dfrac{\pi}{5}}{2}}$

Ex: Complete the following identities.

1) $\tan\dfrac{\theta}{2}\csc\theta =$, 2) $\dfrac{\tan\dfrac{\theta}{2}}{\tan\theta} =$, 3) $\dfrac{1-\cos\theta}{\sin^2\dfrac{\theta}{2}}$