Chapter 5:   Angles and Arcs

## 5.1:   Angles: Set of points determined by two rays.

Angles
- Positive
- Negative
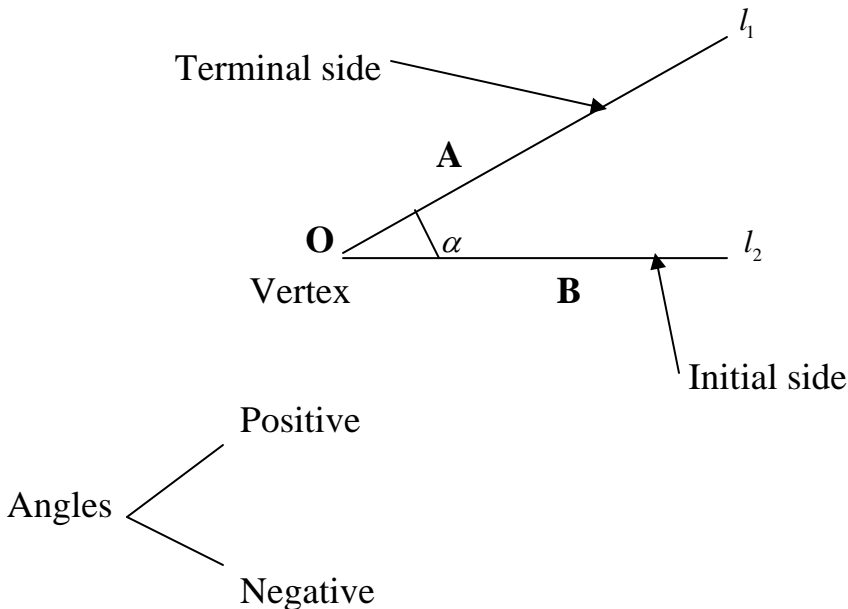
Notes:
1) One unit of measurement for angles is the degree.
2) Angle in standard position obtained by one complete revolution in the counterclockwise direction has measure 360 degree (written 360°)
3) A right angle is a 90° and equal to $\dfrac{1}{4}$ revolution
4) A straight angle is a 180° and equal to $\dfrac{1}{2}$ revolution
5) Quadrantal angles: 90°, 180°, 270°, …..
6) $1^o = \dfrac{1}{360}$ revolution

EX: Draw each angle
   a) 60° , b) -45° , c) 225° , d) -210° , e) 405°

Radian: A central angle has a measure 1 radian if it intercepts an arc with length equal to the radius of the circle.

Arc length = r
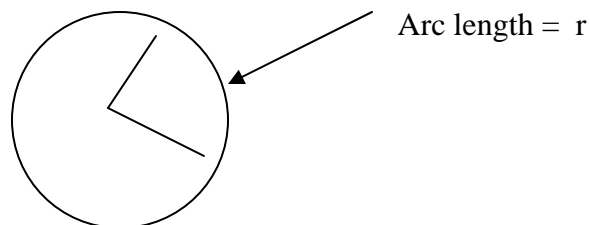
Notes:
1) 2π radian $= 1$ revolution $= 360^o$
2) π radian $= 180^o$

3) 1 radian $= (\frac{180}{\pi})^o \approx 57.2958^o$

4) $1^o = \frac{\pi}{180}$ radian ≈ 0.0174533 radian

Notes:

1) To change radians to degree, multiply by $\frac{180}{\pi}$

2) To change degrees to radian, multiply by $\frac{\pi}{180}$

EX: Convert the following angles to degree measures

a) $-\frac{\pi}{3}$ rad.   b) $\frac{3\pi}{4}$ rad.   c) $-\frac{5\pi}{6}$ rad.

EX: Convert the following angles to radian measures

a) 210° b) -405°

Arc Length Formula:
If an arc of length S on a circle of radius r subtends a central angle of radian measure θ then

$$S = r\theta$$

EX: Arc Length:

1) A central angle θ is subtended by an arc 10 cm long on a circle of diameter 8 cm. Find the measure of θ in   a) radian   b) degree

2) Find S given $r = 3$ ft , $\theta = \frac{7\pi}{2}$

3) Find S given $r = 5$ ft , $\theta = 144^o$

4) Find the number of radians in $\frac{3}{8}$ revolution

5) A bike has wheels that are 28 inches in diameter. How far does the bike move as wheels roll through an angle of 15°

Area of a Circular Sector:

$$A = \frac{1}{2}r^2\theta$$

$r =$ **radius**
$\theta =$ **central angle measures in radians.**

EX: <u>Area of a circular Sector:</u>

1) If $\theta = 50^o$, $r = 8\,m$ Find a) S b) A

2) Find the area of a sector of a circle of diameter 8 ft formed by an angle of 30°

3) Find the area of a circular sector with central angle $1/4$ revolution if the length of intercepted arc is $\dfrac{4\pi}{3}$ centimeters.

3) The area of a sector of a circle with radius 3 centimeters is $\dfrac{3\pi^2}{4}$ square centimeters. Find the length of The intercepted arc in centimeters.
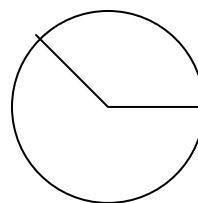

<u>Circular Motion:</u>

If an object moves in a circular path , two speeds are involved.
1) **The rate at which distance is traveled along the circle, called linear speed** v
2) **The rate at which the object revolves about the center of the circle, called angular speed** w

$v =$ **(length/time)**
$w =$ **(radian/time)**

Also $v =$

<u>Notes:</u>
1) **The time units in v and w must be the same**
2) **The linar units used in v and r must be the same**

<u>Ex:</u>
1) A wheel is rotating at 200 revolutions per minute. Find the angular speed in radians per minute
2) An object is traveling around a circle with a radius of 2 m. If in 20 seconds the object travels 5 m , what is its angular speed? what is its linear speed?
3) A rock is spinning at 180 rpm at the end of a 2-foot rope. Find the rock's linear speed (in ft/min).
4) The windshield wiper of a car is 18 inches long. How many inches will the tip of the wiper move during $\dfrac{1}{3}$ revolution.

5) A pendulum swings through an angle of 15° each second. If the pendulum is 20 inches long, how many inches does its tip move each second?

6) Find the radius (in feet) of a circle, if a central angle of 95° subtends a 95 foot arc.