Section 8.3 Exercises:

I) Convert to polar form $z = r(\cos\theta + i\sin\theta)$.

1) $2-2\sqrt{3}\,i$, 2) $-4\sqrt{3}+4\,i$, 3) $-2-2\,i$, 4) $-5\sqrt{3}-5\,i$

5) $-6+6\sqrt{3}\,i$, 6) $-\sqrt{3}\,i$, 7) -5 , 8) $8\,i$

II) Convert to rectangular form (standard form) $z = x+iy$

1) $\sqrt{2}(\cos(-\frac{5\pi}{4})+i\sin(-\frac{5\pi}{4}))$, 2) $6(\cos(\frac{2\pi}{3})+i\sin(\frac{2\pi}{3}))$

3) $3(\cos(-\frac{3\pi}{2})+i\sin(-\frac{3\pi}{2}))$, 4) $4(\cos(\frac{11\pi}{6})+i\sin(\frac{11\pi}{6}))$

III) For the following complex numbers in polar form, z and w , find the standard form (x+iy) of zw , z/w, and w/z

1) $z = 2[\cos(-\frac{5\pi}{6})+i\sin(-\frac{5\pi}{6})]$, $w = 4[\cos(-\frac{\pi}{2})+i\sin(-\frac{\pi}{2})]$

2) $z = 6[\cos(-\frac{5\pi}{12})+i\sin(-\frac{5\pi}{12})]$, $w = 3[\cos(-\frac{11\pi}{12})+i\sin(-\frac{11\pi}{12})]$

3) $z = 4[\cos(-\frac{7\pi}{8})+i\sin(-\frac{7\pi}{8})]$, $w = 2[\cos(\frac{5\pi}{8})+i\sin(\frac{5\pi}{8})]$

4) $z = 3[\cos(-\frac{13\pi}{12})+i\sin(-\frac{13\pi}{12})]$, $w = 6[\cos(-\frac{\pi}{12})+i\sin(-\frac{\pi}{12})]$

IV) For the following complex numbers in standard form, z and w , find the polar form $r(\cos\theta + i\sin\theta)$ of zw , z/w, and w/z

1) $z = -1+\sqrt{3}\,i$, $w = \sqrt{3}-i$ 2) $z = \sqrt{2}+\sqrt{2}\,i$, $w = -1-i$

3) $z = -3\,i$, $w = -2\sqrt{3}-2i$ 4) $z = -6$, $w = 4-4\sqrt{3}\,i$

V) Use De Moivre's theorem to evaluate the following. Express the answer in Standard form x+iy.

1) $\left(-\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)^{12}$, 2) $\left(-1+\sqrt{3}\,i\right)^{7}$, 3) $\left(\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i\right)^{13}$, 4) $\left(\sqrt{3}-i\right)^{5}$

5) $[3(\cos 240^{o}+i\sin 240^{o})]^{4}$, 6) $[2(\cos 112.5^{o}+i\sin 112.5^{o})]^{6}$

7) $\left(\cos\frac{7\pi}{18}+i\sin\frac{7\pi}{18}\right)^{9}$, 8) $\left(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right)^{10}$

9) $[2(\cos\frac{\pi}{9}+i\sin\frac{\pi}{9})]^{6}$, 10) $(\cos 1.35^{o}+i\sin 1.35^{o})^{100}$

VI) Find the indicated roots and express them in standard form $x+iy$ if they are exact roots; otherwise leave them in polar form $r(\cos\theta+i\sin\theta)$

1) square roots of $9i$, 2) fourth roots of $-8+8\sqrt{3}\,i$, 3) cubic roots of $-64i$

4) sixth root of -1 , 5) cubic roots of $4\sqrt{3}-4\,i$

6) square root of $64[\cos(-\frac{\pi}{3})+i\sin(-\frac{\pi}{3})]$

7) fourth root of $81[\cos(\frac{\pi}{8})+i\sin(\frac{\pi}{8})]$

VII) Evaluate the following and express the answer in standard form $x+iy$

1) If $z=\sqrt{2}[\cos(-\frac{5\pi}{8})+i\sin(-\frac{5\pi}{8})]$ and

$w=\sqrt{8}[\cos(-\frac{\pi}{12})+i\sin(-\frac{\pi}{12})]$, find $z^{4}w^{2}$ *and* $\frac{w^{2}}{z^{4}}$

2) If $z=2[\cos(-\frac{7\pi}{36})+i\sin(-\frac{7\pi}{36})]$ and

$w=\cos(-\frac{\pi}{20})+i\sin(-\frac{\pi}{20})$, find $z^{3}w^{5}$ *and* $\frac{z^{3}}{w^{5}}$

3) $z = 2[\cos(\frac{11\pi}{24}) + i\sin(\frac{11\pi}{24})]$ and

$w = \sqrt{2}[\cos(-\frac{\pi}{32}) + i\sin(-\frac{\pi}{32})]$, find $z^2 w^8$ *and* $\frac{w^8}{z^2}$

4) $z = \sqrt[3]{3}[\cos(-\frac{7\pi}{36}) + i\sin(-\frac{7\pi}{36})]$ and

$w = \sqrt[5]{3}[\cos(\frac{7\pi}{30}) + i\sin(\frac{7\pi}{30})]$, find $z^{12} w^5$ *and* $\frac{z^{12}}{w^5}$

5) $z = \sqrt[3]{3}[\cos(-\frac{5\pi}{54}) + i\sin(-\frac{5\pi}{54})]$ and

$w = \sqrt{3}[\cos(\frac{5\pi}{12}) + i\sin(\frac{5\pi}{12})]$, find $z^9 w^4$ *and* $\frac{z^9}{w^4}$

VIII) Evaluate the following and express the answer in polar form $r(\cos\theta + i\sin\theta)$ and in standard form $x + iy$

1) $(2\sqrt{3} - 2i)^{-3}$, 2) $\frac{(-2+2i)^4}{(1+i)^3}$, 3) $\frac{(1-\sqrt{3}\,i)^3}{(-\sqrt{3}+i)^4}$

4) $(-\frac{1}{2} + \frac{1}{2}i)^{-4}$, 5) $(\sqrt{3} - i)^3\,(-1+i)^4$, 6) $(-1+\sqrt{3}i)^4\,(2+2i)^2$

IX) Convert the following polar forms to standard forms $x + iy$

1) $26\,[\cos\{\tan^{-1}(\frac{12}{5})\} + i\sin\{\tan^{-1}(\frac{12}{5})\}]$

2) $5\,[\cos\{\tan^{-1}(-\frac{3}{4})\} + i\sin\{\tan^{-1}(-\frac{3}{4})\}]$

3) $\sqrt{2}\,[\cos\{\pi - \tan^{-1}(\sqrt{8})\} + i\sin\{\pi - \tan^{-1}(\sqrt{8})\}]$

4) $\sqrt{3}\,[\cos\{\tan^{-1}(\frac{\sqrt{6}}{\sqrt{3}}) - 2\pi\} + i\sin\{\tan^{-1}(\frac{\sqrt{6}}{\sqrt{3}}) - 2\pi\}]$

5) $12\,[\cos\{\pi+\tan^{-1}(-\frac{5}{\sqrt{11}})\}+i\,\sin\{\pi+\tan^{-1}(-\frac{5}{\sqrt{11}})\}]$

6) $\cos\{-\pi-\tan^{-1}(\frac{8}{15})\}+i\,\sin\{-\pi-\tan^{-1}(\frac{8}{15})\}$

7) $10\,[\cos\{\frac{\pi}{2}+\tan^{-1}(\frac{4}{3})\}+i\,\sin\{\frac{\pi}{2}+\tan^{-1}(\frac{4}{3})\}]$

8) $8\,[\cos\{\frac{3\pi}{2}-\tan^{-1}(\frac{3}{\sqrt{7}})\}+i\,\sin\{\frac{3\pi}{2}-\tan^{-1}(\frac{3}{\sqrt{7}})\}]$

9) $25\,[\cos\{\tan^{-1}(-\frac{7}{24})-\frac{\pi}{2}\}+i\,\sin\{\tan^{-1}(-\frac{7}{24})-\frac{\pi}{2}\}]$

10) $\cos\{\tan^{-1}(-\frac{15}{8})-\frac{7\pi}{2}\}+i\,\sin\{\tan^{-1}(-\frac{15}{8})-\frac{7\pi}{2}\}$

X) Evaluate the following and express the answer in standard form $x+iy$

1) If $z=-243i$, find the value of $z^{\frac{1}{5}}$ when $k=3$

2) If $z=-2+2\sqrt{3}\,i$, find the value of $z^{\frac{1}{4}}$ when $k=2$

3) If $z=729$, find the value of $z^{\frac{1}{6}}$ when $k=5$

4) If $z=-512$, find the value of $z^{\frac{1}{9}}$ when $k=7$

5) If $z=512i$, find the value of $z^{\frac{1}{9}}$ when $k=5$

6) 6) If $z=\sqrt{512}+\sqrt{512}\,i$, find the value of $z^{\frac{1}{5}}$ when $k=3$