Sections 6.2 and 6.3

I) Complete each equation to get a specific case of an addition or subtraction or double angle or half angle formula.

1) $\sin 5^\circ \cos 12^\circ + \cos 5^\circ \sin 12^\circ = ?$ 2) $\cos\dfrac{3\pi}{10}\cos\dfrac{\pi}{5} + \sin\dfrac{3\pi}{10}\sin\dfrac{\pi}{5} = ?$ 3) $\dfrac{\tan(-15^\circ) - \tan(-20^\circ)}{1 + \tan(-15^\circ)\tan(-20^\circ)} = ?$

4) $\sin(\theta + 5^\circ)\cos(\theta - 5^\circ) + \cos(\theta + 5^\circ)\sin(\theta - 5^\circ) = ?$ 5) $\cos(\dfrac{x}{2} - 1)\cos(\dfrac{x}{2} + 1) - \sin(\dfrac{x}{2} - 1)\sin(\dfrac{x}{2} + 1) = ?$

6) $2\sin 10^\circ \cos 10^\circ = ?$ 7) $1 - 2\sin^2 5\theta = ?$ 8) $\cos^2 3\theta - \sin^2 3\theta = ?$ 9) $2\cos^2\dfrac{\theta}{2} - 1 = ?$

10) $\sqrt{\dfrac{1 - \cos 20^\circ}{2}} = ?$ 11) $\dfrac{2\tan\dfrac{\pi}{14}}{1 - \tan^2\dfrac{\pi}{14}} = ?$ 12) $\dfrac{1 - \cos\dfrac{\pi}{5}}{\sin\dfrac{\pi}{5}} = ?$ 13) $-\sqrt{\dfrac{1 + \cos 230^\circ}{2}} = ?$

II) Find the exact value for each expression.

1) $\cos 23^\circ \cos 37^\circ - \sin 23^\circ \sin 37^\circ$,   2) $\sin 54^\circ \cos 9^\circ - \cos 54^\circ \sin 9^\circ$,   3) $\dfrac{\tan(45^\circ) - \tan(75^\circ)}{1 + \tan(45^\circ)\tan(75^\circ)}$

4) $\cos\dfrac{2\pi}{9}\cos\dfrac{5\pi}{9} + \sin\dfrac{2\pi}{9}\sin\dfrac{5\pi}{9}$,   5) $\dfrac{\tan(-4^\circ) + \tan(-56^\circ)}{\tan(-4^\circ)\tan(-56^\circ) - 1}$,   6) $\dfrac{\tan(\pi) - \tan(\dfrac{9\pi}{12})}{1 + \tan(\pi)\tan(\dfrac{9\pi}{12})}$

III) Given $\sec\alpha = -3$, $\tan\alpha < 0$ *and* $\cot\beta = \sqrt{2}$, $\csc\beta < 0$ find the exact value for

a) $\sin(\alpha + \beta)$ b) $\tan(\alpha - \beta)$ c) $\cos(\alpha + \beta)$ d) $\tan\dfrac{\alpha}{2}$ e) $\sec\dfrac{\alpha}{2}$ f) $\sin\dfrac{\beta}{2}$ g) $\cos\dfrac{\beta}{2}$

h) $\cos 2\alpha$ i) $\sin 2\beta$ j) $\tan 2\alpha$ $\left[note : 0 < \alpha, \beta < 2\pi\right]$

IV) Use the addition or subtraction formulas to find the exact value of $\sin\theta$, $\cos\theta$, *and* $\tan\theta$ for the specified value of θ

1) $\theta = 105^\circ$ 2) $\theta = 15^\circ$ 3) $\theta = 165^\circ$ 4) $\theta = \dfrac{23\pi}{12}$ 5) $\theta = 285^\circ$ 6) $\theta = \dfrac{5\pi}{12}$ 7) $\theta = \dfrac{17\pi}{12}$

V) Use the addition or subtraction formulas to verify the identity.

1) $\cos(\theta + 30^\circ) + \cos(\theta - 30^\circ) = \sqrt{3}\cos\theta$ 2) $\cos(\theta + \dfrac{\pi}{6}) + \sin(\theta - \dfrac{\pi}{3}) = 0$ 3) $\dfrac{\sin(\theta + \dfrac{\pi}{2})}{\cos(\theta + \dfrac{\pi}{2})} = -\cot\theta$

4) $\cos(\alpha + \beta)\cos(\alpha - \beta) = \cos^2\alpha - \sin^2\beta$ 5) $\cos\alpha\cos(\alpha - \beta) + \sin\alpha\sin(\alpha - \beta) = \cos\beta$

VI) Use half angle formulas to find the exact value for the following

1) $\sin\dfrac{5\pi}{12}$ 2) $\tan 67.5^\circ$ 3) $\cos 112.5^\circ$ 4) $\sin\dfrac{7\pi}{8}$ 5) $\cot\dfrac{\pi}{12}$ 6) $\sin(-\dfrac{\pi}{8})$ 7) $\cos(-165^\circ)$

VII) Use the double angle formulas to find the exact value for the following

1) $\cos^2\dfrac{\pi}{12} - \sin^2\dfrac{\pi}{12}$ 2) $1 - 2\sin^2\dfrac{5\pi}{8}$ 3) $\sin 105^\circ \cos 105^\circ$ 4) $1 - 2\cos^2 15^\circ$ 5) $\dfrac{\tan 22.5^\circ}{1 - \tan^2 22.5^\circ}$

VIII) If $\sin\theta = -\dfrac{\sqrt{3}}{3}$, $\dfrac{3\pi}{2} < \theta < 2\pi$ find

1) $\sin 2\theta$ 2) $\cos 2\theta$ 3) $\tan 2\theta$ 4) $\sin\dfrac{\theta}{2}$ 5) $\cos\dfrac{\theta}{2}$ 6) $\tan\dfrac{\theta}{2}$

IX) 1) Given $-2\pi < \theta < 0$. If $\sin\theta = -\dfrac{4}{5}$, and $\tan\theta > 0$ find $\cos\dfrac{\theta}{2}$

2) Given $-2\pi < \theta < 0$. If $\tan\theta = \dfrac{3}{4}$, and $\cos\theta > 0$ find $\sin\dfrac{\theta}{2}$

X) Use the double angle and the half angle formulas to verify the identity

1) $\dfrac{\sin\theta + \tan\theta}{2\tan\theta} = \cos^2\dfrac{\theta}{2}$, 2) $\dfrac{2}{\cot\theta - \tan\theta} = \tan 2\theta$, 3) $\dfrac{\sin 2\theta - \sin\theta}{\cos 2\theta + \cos\theta} = \tan\dfrac{\theta}{2}$

4) $\sin\theta\sec\dfrac{\theta}{2} = 2\sin\dfrac{\theta}{2}$, 5) $\csc\theta = \cot\theta + \tan\dfrac{\theta}{2}$, 6) $\dfrac{\sec^2\theta}{2 - \sec^2\theta} = \sec 2\theta$

XI) 1) Find $\tan\theta$ if $\cos 2\theta = -\dfrac{1}{8}$, $\dfrac{\pi}{2} < \theta < \pi$

2) Find $\sec\theta$ if $\tan 2\theta = -\dfrac{8}{15}$, $\dfrac{3\pi}{2} < \theta < 2\pi$

XII) Evaluate the following using addition or subtraction or double angle or half angle
Formulas

1) $\cos\dfrac{\pi}{10}\cos\dfrac{2\pi}{5} + \sin\dfrac{\pi}{10}\sin\dfrac{2\pi}{5} =$

2) $\sin(-5^\circ)\cos(-7^\circ) - \sin(-7^\circ)\cos(-5^\circ) =$

3) $\dfrac{\tan(-75^\circ) + \tan(35^\circ)}{1 + \tan 75^\circ \tan 35^\circ} =$

4) $\sin(-10^\circ)\cos(-10^\circ) =$

5) $1 - 2\sin^2 3\theta =$

6) $\sin^2\dfrac{\theta}{6} - \cos^2\dfrac{\theta}{6} =$

7) $2\cos^2\dfrac{\theta}{3} - 1 =$

8) $\sqrt{\dfrac{1 + \cos(-25^\circ)}{2}} =$

9) $\dfrac{\tan\dfrac{\pi}{8}}{1 - \tan^2\dfrac{\pi}{8}} =$

10) $\dfrac{1 - \cos\dfrac{\pi}{11}}{\sin\dfrac{\pi}{11}} =$

11) $\dfrac{\sin\dfrac{7\pi}{5}}{1 + \cos\dfrac{7\pi}{5}} =$

12) $\sqrt{\dfrac{1 - \cos(-\dfrac{3\pi}{7})}{2}} =$

XIII) 1) If $\sin\theta = -\dfrac{3}{5}$ then $\cos(\dfrac{3\pi}{2} - \theta) =$

2) If $\sec\theta = -\dfrac{3}{2}$ then $\csc(\dfrac{3\pi}{2} + \theta) =$

3) If $\tan\theta = -3$ then $\cot(\dfrac{\pi}{2} + \theta) =$

4) If $\cot\theta = -\dfrac{1}{3}$ then $\tan(\dfrac{3\pi}{2} - \theta) =$

5) If $\csc\theta = -3$ then $\cos(\dfrac{\pi}{2} + \theta) =$

XIV) Find the exact value of each expression

1) $\tan[\dfrac{1}{2}\sin^{-1}(-1)]$, **2)** $\cos(2\cos^{-1}\dfrac{1}{2})$, **3)** $\sin[\dfrac{1}{2}\tan^{-1}(-\sqrt{3})]$, **4)** $\sin[2\cos^{-1}(-\dfrac{\sqrt{3}}{2})]$

5) $\tan[\dfrac{1}{2}\cos^{-1}(-1)]$, **6)** $\cos(2\sin^{-1}\dfrac{\sqrt{3}}{2})$, **7)** $\sin[2\cos^{-1}(-\dfrac{1}{\sqrt{2}})]$, **8)** $\cos(2\cos^{-1}0)$, **9)** $\tan[\dfrac{1}{2}\sin^{-1}(-\dfrac{4}{5})]$

10) $\csc[2\tan^{-1}(-\dfrac{3}{4})]$, **11)** $\sec(2\sin^{-1}\dfrac{3}{4})$, **12)** $\sin(\dfrac{1}{2}\cos^{-1}\dfrac{4}{5})$, **13)** $\cos[\dfrac{1}{2}\sin^{-1}(-\dfrac{12}{13})]$

14) $\cos[\tan^{-1}(-1) + \cos^{-1}(-\dfrac{4}{5})]$, **15)** $\tan[\sin^{-1}(-\dfrac{3}{5}) - \dfrac{\pi}{4}]$, **16)** $\csc[\tan^{-1}(-\dfrac{4}{3}) + \dfrac{7\pi}{2}]$,

17) $\tan[\dfrac{11\pi}{2} - \cos^{-1}(-\dfrac{12}{13})]$, **18)** $\cos[\sin^{-1}(-\dfrac{3}{5}) - 5\pi]$, **19)** $\cot[\dfrac{5\pi}{2} + \cos^{-1}(\dfrac{1}{3})]$

20) $\sin[4\pi - \tan^{-1}(2\sqrt{2})]$

XV) Complete the following identities

1) $4\sin(\theta - \dfrac{\pi}{3})\cos(\theta - \dfrac{\pi}{6}) =$

2) $\dfrac{\tan(\theta - \dfrac{3\pi}{2}) - \cot(6\pi + \theta)}{2\sin(\dfrac{\pi}{2} + \theta)} =$

3) $\dfrac{\csc(\dfrac{3\pi}{2} - \theta)}{\sec(\dfrac{\pi}{2} + \theta)} =$

4) $\tan(\dfrac{5\pi}{4} - \theta)\tan(\dfrac{7\pi}{4} - \theta) =$