

## 1. SECTION 5.4 SOLVING POLYNOMIAL AND RATIONAL INEQUALITIES

### The Sign Chart Method (usually taught in College Algebra)

Step 1. Add or subtract so that all terms of the inequality are on one side and 0 is on the other side. You need to get the equation in one of the following forms:

$$\begin{array}{ll} f(x) > 0 & f(x) \geq 0 \\ f(x) < 0 & f(x) \leq 0 \end{array}$$

Step 2. Simplify the expression  $f(x)$ . Combine and factor.

Step 3. Find where  $f(x)$  is equal to 0 or undefined (i.e. where is the numerator and where is the denominator equal to zero). We will call these values the **Partitioning Values**.

Step 4. Put these numbers on a number line. The Partition values will separate (partition) the number line into one more intervals.

Step 5. Make a chart by making each interval in Step 4 a column and each factor in  $f(x)$  a row. You may combine more than one factor in the rows, but every factor should be accounted for.

Step 6. Select a test number in each interval and determine if the factor in each row is positive or negative.

Step 7. Add a row in your table for  $f(x)$  and determine the sign of  $f(x)$  in each interval (column) by counting the negatives in each column (even number of negatives makes  $f$  positive and an odd number of negatives makes  $f$  negative).

Step 8. Answer the question based on which of the inequalities

$$\begin{array}{ll} f(x) > 0 & f(x) \geq 0 \\ f(x) < 0 & f(x) \leq 0 \end{array}$$

you are to solve and the signs given for  $f$  in your table.

**Remark 1.1.** *This method works because a polynomial or rational function can only change signs where it equals zero or is undefined.*

**Example 1.1.** Solve  $\frac{1}{3-x} \geq 2$ .

**Example 1.2.** Solve  $\frac{4x-1}{(x+2)(x+3)} \leq 0$ .

- (A)  $[-3, -2] \cup [1/4, \infty)$
- (B)  $(-3, -2) \cup [1/4, \infty)$
- (C)  $(-\infty, -3] \cup [-2, 1/4]$
- (D)  $(-\infty, -3) \cup (-2, 1/4]$

**Remark 1.2.** The union symbol in sets is not the same as the letter  $U$  and does not appear on your keyboard. To enter the union in MML you have to use the math tool palette.

## 2. NEW TOOLS IN PRECALCULUS

You have learned tools to quickly sketch a variety of polynomials and rational functions. Sketch the graph of the function and then determine solution to the inequality using the graph.

**Example 2.1.** *Solve  $x^2 - x - 6 > 0$  by sketching the graph.*

We can also use a combination of the sign chart and information about the graphs of functions that we have learned this semester to make the solution quicker to find.

**Example 2.2.** *Solve  $-2(x + 4)^2(x^2 + 9)(4 - x)^3 > 0$ .*

**Example 2.3.** Solve  $\frac{x^4(x^2 + 4)(x^2 - 4)}{(2x + 1)^2(x - 4)^3} \leq 0$ .