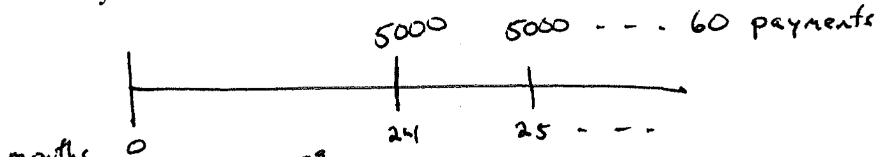
MAP 4170 Name:____________________________
Test 3 Date: March 29, 2022

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. A loan of L is repaid with 60 level monthly payments of 5000 with the first payment due 2 years after the loan inception date. Interest is charged using an interest rate of 9% compounded monthly. Determine L.



(A) 199,825

(B) 201,325

(C) 202,835

(D) 204,355

(E) 205,890

$\text{meir} = \frac{.09}{12} = .0075$

$$L = 5000\,\ddot{a}_{\overline{60}|.0075} \cdot v^{24}_{.0075} = 202834.02$$

2. A 20-year 1000 face value callable bond with 10% semiannual coupons is redeemable at the end of any year starting with year 16. The redemption value schedule is:

   1000 at the end of year 16, 17, or 18
   900 at the end of year 19 or 20

   The bond is bought at the maximum price to guarantee a semiannual effective yield of at least 4%. Determine the yield, as a semiannual effective interest rate, if the bond is called at the end of year 17.

(A) 4.03%

(B) 4.04%

(C) 4.05%

(D) 4.06%

(E) 4.07%

n	$P(0.04)$
32	$50a_{\overline{32}|} + 1000v^{32} = 1178.74$
36	$50a_{\overline{36}|} + 1000v^{36} = 1189.08$
38	$50a_{\overline{38}|} + 900v^{38} = 1171.15$
40	$50a_{\overline{40}|} + 900v^{40} = 1177.10$

Bond is bought at price $P = 1171.15$

If called at the end of year 17, then

$$1171.15 = 50a_{\overline{34}|i} + 1000v_i^{34}$$

$$\Longrightarrow i = .04062\cdots$$

3. A 10-year bond with level semiannual coupons is bought at a price of 1074. The amount for amortization of discount in the 12th coupon payment is 6.51, and the amount for amortization of discount in the 15th coupon payment is 7.75. Determine the bond's redemption value.

(A) 948

(B) 1011

(C) 1074

(D) 1136

(E) 1200

$P_{15} = P_{12} \cdot (1+i)^3 \Rightarrow -7.75 = -6.51 \cdot (1+i)^3 \Rightarrow i = 0.0598\ldots$

$P - R = P_1 \cdot s_{\overline{20}|i}$ $\quad P_1 = P_{12} \cdot v^{11} = -6.51 \cdot v^{11} = -3.435\ldots$

$\Rightarrow 1074 - R = (-3.435\ldots) \cdot s_{\overline{20}|.0598\ldots} = -126.144\ldots$

$\therefore R = 1200.144\ldots$

4. A loan is amortized with level payments at the end of each month for 120 months. The amount of interest paid in the 57th payment is 481.51 and the amount of interest paid in the 89th payment is 269.40. Determine the amount of principal repaid in the 105th payment.

(A) 1104

(B) 1124

(C) 1144

(D) 1164

(E) 1184

$I_{57} = 481.51 = i \cdot B_{56} = i \cdot C a_{\overline{64}|}$

$I_{89} = 269.40 = i \cdot B_{88} = i \cdot C a_{\overline{32}|}$

$i \cdot C a_{\overline{64}|} = i \cdot C \cdot a_{\overline{32}|} \cdot (1 + v^{32})$

$\Rightarrow 481.51 = 269.40 \cdot (1 + v^{32})$

$\Rightarrow v^{32} = \frac{481.51}{269.4} - 1 \Rightarrow i = 0.0075$

$\therefore B_{88} = \frac{269.4}{.0075} = 35920 = C a_{\overline{32}|.0075} \Rightarrow C = 1266.75$

$B_{104} = \overset{1266.75}{C} a_{\overline{16}|.0075} = 19032.30$

$\Rightarrow I_{105} = .0075(19032.30) = 142.74 \Rightarrow P_{105} = C - I_{105}$

$= 1124.01$

5. A loan of 10000 is repaid with 120 payments at the end of each month, using a nominal interest rate of 6% compounded monthly. The first payment is equal to 2% of the loan amount. The second payment is 2% of the outstanding balance remaining after the first payment. The third payment is 2% of the outstanding balance remaining after the second payment, and so on, through the 60th payment. The 61st through the 120th payments are level and equal to X. Determine X.

$i = \text{meir} = \frac{.06}{12} = .005$

(A) 65

$L = B_0 = 10000$

(B) 78

$C_1 = .02 B_0 \quad I_1 = i \cdot B_0 = .005 B_0$

$\therefore P_1 = C_1 - I_1 = .02 B_0 - .005 B_0 = .015 B_0$

(C) 91

(D) 106

$\therefore B_1 = B_0 - P_1 = B_0 - .015 B_0 = .985 B_0$

(E) 121

Continuing, $B_{60} = (.985)^{60} \cdot B_0 = X a_{\overline{60}|}$

$\therefore (.985)^{60} \cdot 10000 = X a_{\overline{60}|.005}$

$\Longrightarrow X = 78.07$

6. A 1000 par value n-year bond maturing at par with annual coupons of 100 is purchased for 1125. The present value of the redemption value is 500.

Find n.

$R = 1000 \Longrightarrow \cancel{500 v^n = 1000}$ $1000 v^n = 500 \Longrightarrow v^n = 0.5$

(A) 6

$1125 = 100 \cdot a_{\overline{n}|} + \underbrace{1000 v^n}_{=500}$

(B) 7

(C) 8

(D) 9

$\therefore 1125 = 100 \cdot \frac{1 - v^n}{i} + 500 \xrightarrow{v^n = 0.5} i = .08$

(E) 10

$1125 = 100 a_{\overline{n}|.08} + 1000 v^n_{.08} \xrightarrow[\text{TVM}]{\text{use}} n = 9$

meir = .01

7. Joe takes out a 30-year mortgage of 200,000 at a nominal interest rate of 12% compounded monthly. He is to repay the loan with level monthly payments using the amortization method. $200000 = C\,a_{\overline{360}|.01} \Rightarrow C = 2057.23$

meir = .0075

At the end of 10 years, interest rates have fallen to 9% compounded monthly. Joe refinances the outstanding balance of his mortgage with this new interest rate using a 10-year level monthly payment amortization. $B_{120} = C\,a_{\overline{240}|.01} = 186835.99$

Determine the total amount of interest Joe saved by refinancing.

(A) 210,000

(B) 220,000

(C) 230,000

(D) 240,000

(E) 250,000

2^{nd} paragraph $\Rightarrow B_{120} = \tilde{C}\,a_{\overline{120}|.0075}$

$\Rightarrow 186835.99 = \tilde{C}\,a_{\overline{120}|.0075} \Rightarrow \tilde{C} = 2366.76$

Without refinancing, Joe pays a total amount of interest equal to $360C - 200000 \approx 540600$

With refinancing, Joe pays a total amount of interest equal to $(120C + 120\tilde{C}) - 200000 \approx 330900$

$\therefore$ Joe saves $540600 - 330900 = 209700$ by refinancing

8. A special 20-year bond, redeemable at 1000, is bought to yield 7.12% annual effective. The bond has increasing annual coupons whereby each coupon is 3% more than its preceding coupon. The initial coupon is 100. Determine the book value of the bond immediately after the 8^{th} coupon is paid.

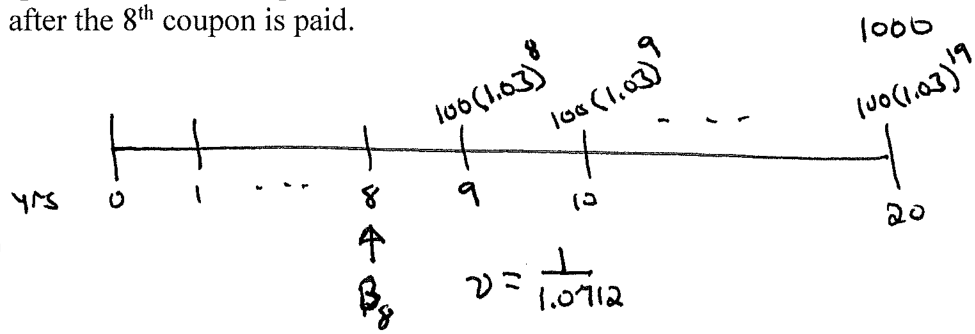
(A) 1533

(B) 1567

(C) 1592

(D) 1627

(E) 1675



$v = \frac{1}{1.0712}$

$$B_8 = 100(1.03)^8 \cdot v + 100(1.03)^9 \cdot v^2 + \ldots 12\text{ terms} + 1000v^{12}$$

$$= \frac{100(1.03)^8}{1.0712} \cdot \left(1 + \frac{1.03}{1.0712} + 12\text{ terms}\right) + 1000v^{12}$$

$$= \frac{100(1.03)^8}{1.0712} \cdot \ddot{a}_{\overline{12}|\left(\frac{1.0712}{1.03} - 1\right)} + 1000v^{12}_{.0712}$$

$$\therefore B_8 = 1592.33$$

9. Jamie buys a 10-year 1000 face value bond, redeemable at par, with 5% semiannual coupons at a price to yield 5% compounded semiannually. Each coupon Jamie receives is deposited into an account that earns a nominal interest rate of 4% compounded semiannually. Immediately after receiving the 8th coupon, Jamie sells the bond to a new investor at a price to yield the new investor 5% compounded semiannually. Determine the yield rate, as a nominal interest rate compounded semiannually, that Jamie earned during the time that she owned the bond.

(A) 2.3%

(B) 2.5%

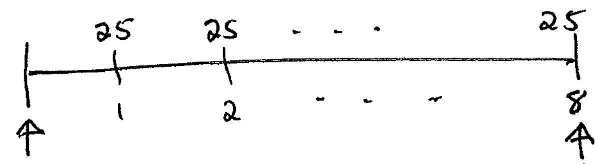
(C) 4.2%

(D) 4.6%

(E) 4.9%

Jamie buys the bond for $P^{buy} = 25a_{\overline{20}|.025} + 1000v^{20}_{.025} = 1000$

Jamie sells the bond for $P^{sell} = 25a_{\overline{12}|.025} + 1000v^{12}_{.025} = 1000$



$P^{buy} = 1000$

$AV = AV_1 + AV_2$

$$\left.\begin{array}{l} AV_1 = 25s_{\overline{8}|.02} \\ AV_2 = P^{sell} = 1000 \end{array}\right\} AV = 25s_{\overline{8}|.02} + 1000 = 1214.57$$

$$\therefore\ 1000\left(1+\frac{i}{2}\right)^8 = 1214.57 \Rightarrow i \approx 0.049$$

10. A loan of 250000 at an annual effective interest rate of 6% is repaid with annual payments. The first payment is 20000 and each subsequent payment is 1000 more than its preceding payment. Determine the amount of principal repaid in the 10th payment.

(A) 19950

(B) 20150

(C) 20350

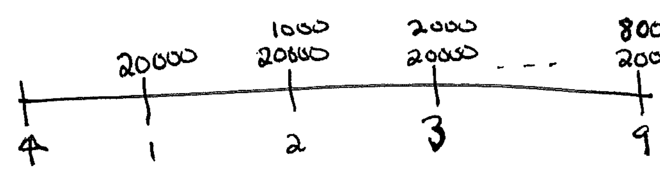
(D) 20550

(E) 20750

$$P_{10} = C_{10} - I_{10} = (20000 + 1000(9)) - .06 \cdot B_9$$

$$= 29000 - .06 \cdot B_9$$

$B_9 = ?$



$L = 250000$

$$\therefore B_9 = 250000(1.06)^9 - \left[20000s_{\overline{9}|.06} + 1000(Is)_{\overline{8}|.06}\right]$$

$$\therefore B_9 = 151021.49$$

$$\therefore P_{10} = 29000 - .06(151021.49) = 19938.71$$