MAC 2313, Section 04 with Dr. Hurdal
Fall 2011 – Test 4

Name: ________________________

As stated in class, you are allowed to bring to the test one 8.5x11 inch page, written on both sides. Calculators are allowed. Notebooks and textbooks are NOT allowed. Marks will be allocated for clear and well written mathematics solutions. This test will be graded out of 100.

1. (20 marks) Use the transformation $T : x = u,\ y = \dfrac{v}{u}$ to evaluate $\displaystyle\int\int_R 2x^3y\,dA$ where R is the region in the first quadrant bounded by the lines $y = x$, $y = 4x$ and the hyperbolas $xy = 1$ and $xy = 5$.

2. (20 marks) Find the work done by the force field $\mathbf{F}(x, y) = \langle xy^2, -x^2 \rangle$ over the curve C where C consists of the parabola $y = x^2$ from $(-1, 1)$ to $(1, 1)$ and the line segment from $(1, 1)$ to $(2, 1)$.

3. (20 marks) Find the volume of the solid that lies within the sphere $x^2 + y^2 + z^2 = 4$, above the xy-plane, and below the cone $z = \sqrt{x^2 + y^2}$.

4. (20 marks) Consider the vector field $\mathbf{F}(x, y) = \langle -y, x \rangle$.
a) Sketch $\mathbf{F}$.
b) On your sketch draw a curve C_1 such that $\int_{C_1} \mathbf{F} \cdot d\mathbf{r} < 0$.
c) If C_2 is a circle of radius 1 centered at the origin and oriented counter clockwise, is $\int_{C_2} \mathbf{F} \cdot d\mathbf{r}$ positive, negative or zero? Explain.
d) Using your result from part c) and without using the curl test, do you think $\mathbf{F}$ is a conservative vector field? Explain.

5. (20 marks) a) Show that $\mathbf{F}(x, y) = \langle 2x - 3y, -3x + 4y - 8 \rangle$ is a conservative vector field.
b) Find a potential function f for $\mathbf{F}$.
c) Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ two different ways where C is the portion of the circle $x^2 + y^2 = 4$ from $(2, 0)$ to $(0, 2)$.

Bonus:
A) (5 marks) Derive the "conversion" factor for spherical coordinates.
B) (5 marks) If $\mathbf{F} = \langle P, Q, R \rangle$ is a vector field on $\mathbb{R}^3$ and the partial derivatives of P, Q and R all exist, then the curl of $\mathbf{F}$ is defined as

$$\text{curl } \mathbf{F} = \nabla \times \mathbf{F}.$$

Recall that $\nabla = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle$. Show that the curl test holds in $\mathbb{R}^3$. That is, show that if $\mathbf{F} = \nabla f$ then curl $\mathbf{F} = \mathbf{0}$.