MAC 2313, Section 04 with Dr. Hurdal
Fall 2011 – Test 3

Name: ________________________

As stated in class, you are allowed to bring to the test one 8.5x11 inch page, written on both sides. Calculators are allowed. Notebooks and textbooks are NOT allowed. Marks will be allocated for clear and well written mathematics solutions. This test will be graded out of 100.

1. (10 marks) Use integration to find the volume bounded by the surface $z = \sqrt{4 - x^2 - y^2}$ and the plane $z = 0$.

2. (20 marks) For the function $f(x, y) = x^3 + y^3 + 3x^2 + 3y^2 - 8$,
a) find all critical points of $f(x, y)$ and use the second derivative test to classify them;
b) what is the maximum rate of change of $f(x, y)$ at the point $(1, -1)$;
c) in which direction does $f(x, y)$ change most rapidly?

3. (10 marks) Evaluate $\int_0^{\sqrt{\pi}/2} \int_x^{\sqrt{\pi}/2} 2y^2 \sin(xy)\, dy\, dx$ by reversing the order of integration.

4. (15 marks) Use Lagrange multipliers to find the points on the cone $z^2 = x^2 + y^2$ that are closest to the point $(4, 2, 0)$.

5. (20 marks) The temperature at a point (x, y) is given by $T(x, y) = \sqrt{x^2 + 3y^2}$ where T is measured in °C and x, y are in meters.

a) Estimate the average temperature in the region $R = [0, 6] \times [0, 6]$ using sample points to be lower right corners with $m = 2$ and $n = 3$.

b) Where is $T(x, y)$ differentiable?

c) Find the rate of change of temperature at the point $(2, -2)$ in the direction toward the point $(3, 0)$.

d) If $x = e^s \cos(t)$ and $y = e^s \sin(t)$, use the chain rule to find $\dfrac{\partial T}{\partial t}$.

6. (10 marks) A boundary stripe 6 cm wide is painted around a rectangle whose dimensions are 100 m by 200 m. Use differentials to approximate the number of square meters of paint in the stripe. Warning: be careful with your units.

7. (15 marks) If two streets forming the boundary a suburb D are given by the parabolas $8y = x^2$ and $x = y^2$, and the population density function of D is given by $\rho(x, y) = \sqrt{x}$ in thousands/km^2, find the total population of D.

Bonus (10 marks): Assuming all the following integrals exist, the following integral property holds:

If $m \leq f(x,y) \leq M$ for all (x,y) in D, then

$$mA(D) \leq \int\int_D f(x,y)\,dA \leq MA(D)$$

where $A(D) = \displaystyle\int\int_D dA$.

Use this property to determine if the statement

$$\int_1^4 \int_0^1 (x^2 + \sqrt{y}) \sin(x^2 y^2)\,dx\,dy \leq 9$$

is true or false. If the statement is true, explain why. If it is false, explain why or give an example that disproves this statement.