MAC 2313, Section 04 with Dr. Hurdal
Fall 2011 – Test 1

Name: ________________________

Calculators are allowed. Notebooks and textbooks are NOT allowed. There will be 10 marks allocated for clear and well written mathematics solutions. This test will be graded out of 100.

1. (20 marks) (a) Sketch at least four traces in one plane (your choice) of the surface given by $y^2 = 2x^2 + z^2$.
(b) Sketch and describe the 3D surface.

2. (10 marks) Find the values of x such that the vectors $\langle 3, 2, x \rangle$ and $\langle 2x, 4, x \rangle$ are orthogonal.

3. (20 marks) Consider the two planes $3x - 6y - 2z = 15$ and $2x + y - 2z = 5$.
a) Sketch the second plane.
b) At what angle do the planes intersect?
c) Find the parametric equations for the line of intersection of the planes.
d) Find the equation of a new plane that contains the line of intersection and the point $(1, 0, -1)$.

4. (10 marks) A plane is flying due east at 500km/hr and experiences a 70 km/hr tailwind blowing in the direction N30°E. What is the new heading and groundspeed the plane needs to fly in order to maintain a compass heading due east?

5. (10 marks) Consider the points $P(2,4,5)$, $Q(1,5,7)$, $R(-1,6,8)$ and $S(-2,7,9)$.
a) Find the volume of the parallelepiped formed by those points.
b) Find the scalar and vector projections of $\vec{PQ}$ onto $\vec{PR}$.

6. (10 marks) Show that the line $x = 1 - 2t$, $y = 2 + 5t$, $z = -3t$ is NOT parallel to the plane $-2x + 5y = 8 + 3z$. Then find the point at which the line intersects the plane.

7. (10 marks) Show that the lines
L_1: $x = 1 + t, y = -2 + 3t, z = 4 - t$ and
L_2: $\vec{r} = \langle 2s, 3 + s, -3 + 4s \rangle$
are skew.

Bonus (10 marks): If $\vec{a} = \langle a_1, a_2, a_3 \rangle$ and $\vec{b} = \langle b_1, b_2, b_3 \rangle$, use the definitions of cross product, dot product and length of a vector to show

$$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$$

for $0 \leq \theta \leq \pi$.