MAC 2313, Section 04 with Dr. Hurdal Fall 2011 – Assignment 2

Due: Wednesday October 19, 2011 at the beginning of class.

With your group (3-4 people per group), please hand in complete written solutions (1 solution set per group in one hand writing) for the following questions. A different scribe must be used for this assignment. Points will be allocated for clear and well written mathematical solutions.

In addition to your group solutions, you will individually hand in a typed log which you sign, that includes when your group met and who was there, when you worked on the problems by yourself and who wrote up the final assignment solutions. Your log should also rank each member of your group (including yourself) with a percentage contribution to the assignment. This log will be kept confidential by me so if you have any group dynamic concerns, then this is a place where you can write your comments. Individuals who do not submit a typed log will have points deducted. It is not guaranteed that every group member will get the same grade.

Homework must be stapled to be accepted.

1. Find the absolute maxima and minima of the function $T(x, y) = x^2 + xy + y^2 - 6x + 2$ on the rectangular plate $0 \le x \le 5, -3 \le y \le 0$.

2. The density ρ (in g/cm³) of carbon dioxide gas CO₂ depends upon its temperature T (in °C) and pressure P (in atmospheres). The ideal gas model for CO₂ gives what is called the state equation

$$\rho(T, P) = \frac{0.5363P}{T + 273.15}.$$

If the temperature is 20°C and the pressure is 2.5 atmospheres, use differentials to estimate the maximum error in density if there is a possible error of 2% in both temperature and pressure measurements.

3. a) Find the dimensions of the rectangle of largest perimeter that can be inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with sides parallel to the coordinate axes.

b) What is the largest perimeter?

c) Use your results from a) and b) and discuss/interpret your answers for the case when a = b.

Bonus: For a curve $\mathbf{r}(t)$, the osculating plane is determined by the vectors \mathbf{T} and \mathbf{N} , so the normal vector for the osculating plane is $\mathbf{T} \times \mathbf{N} = \mathbf{B}$. In class I discussed that the normal vector for the osculating plane can also be given by $\mathbf{r}'(t) \times \mathbf{r}''(t)$. Show that this is true.