MAC 2313, Section 04 with Dr. Hurdal
Spring 2008 Final Exam Answers

1. 12

2. $\frac{1}{12}(1-\cos 1)$

3. 64

4. The point is $\left(\frac{1}{6}, \frac{1}{3}, \frac{1}{6}\right)$ and the distance of this point from the origin is $\frac{1}{\sqrt{6}} \approx 0.4082$ units.

5. a) Level curves occur for $k \geq 0$ and are ellipses with minor axis along the x-axis and major axis along the y-axis.
b) $z = 6x + 8y - 5$
c) $2\sqrt{5}$

6. Compute flux directly AND using the divergence theorem. Total flux is 24π. Computing flux directly requires summing the computations of the flux on the paraboloid part of the surface which is 8π and the flux on the disk part of the surface which is 16π.

7. -16

Bonus: a) Use the definition of curl and divergence to prove part a). By definition,

$$\text{curl } \mathbf{F} = \left\langle \frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}, -\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right), \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right\rangle.$$

Then,

$$\begin{aligned}
\text{div}(\text{curl } \mathbf{F}) &= \text{div} \left\langle \frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}, -\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right), \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right\rangle \\
&= \frac{\partial}{\partial x}\left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right) + \frac{\partial}{\partial y}\left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) \\
&= \frac{\partial^2 R}{\partial x\,\partial y} - \frac{\partial^2 Q}{\partial x\,\partial z} + \frac{\partial^2 P}{\partial y\,\partial z} - \frac{\partial^2 R}{\partial y\,\partial x} + \frac{\partial^2 Q}{\partial z\,\partial x} - \frac{\partial^2 P}{\partial z\,\partial y} \\
&= 0 \quad \text{since } \frac{\partial^2 R}{\partial x\,\partial y} = \frac{\partial^2 R}{\partial y\,\partial x}, \frac{\partial^2 Q}{\partial z\,\partial x} = \frac{\partial^2 Q}{\partial x\,\partial z}, \frac{\partial^2 P}{\partial y\,\partial z} = \frac{\partial^2 P}{\partial z\,\partial y} \text{ by Clairaut's Theorem.}
\end{aligned}$$

b) Since $\int\int_S \mathbf{F} \cdot d\mathbf{S} = \int\int\int_E \text{div } \mathbf{F}\, dV$ by the Divergence Theorem, then

$$\begin{aligned}
\int\int_S \text{curl } \mathbf{F} \cdot d\mathbf{S} &= \int\int\int_E \text{div}(\text{curl } \mathbf{F})\, dV \\
&= \int\int\int_E 0\, dV \quad \text{since div(curl}\mathbf{F}) = 0 \text{ from part a)} \\
&= 0 \quad \text{as required.}
\end{aligned}$$