MAC 2313, Section 04 with Dr. Hurdal
Spring 2008 – Final Exam

Name: ________________________

As stated in class, you are allowed to bring to the test one 8.5x11 inch page, written on both sides. Calculators are allowed. Notebooks and textbooks are NOT allowed.

1. (15 marks) Use Stokes' Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} =< -18y, z^2, e^z >$ and C is the boundary of the part of the plane in the first octant oriented counterclockwise with vertices $(1, 0, 0)$, $(0, 2, 0)$ and $(0, 0, 3)$.

2. (10 marks) Sketch the region, change the order of integration and evaluate $\int_0^1 \int_{x^2}^1 x^3 \sin(y^3)\, dy\, dx$.

3. (10 marks) Directly evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is given by the vector function $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$, $0 \leq t \leq 2$ and $\mathbf{F}(x, y, z) = yz\mathbf{i} + xz\mathbf{j} + xy\mathbf{k}$.

4. (10 marks) Use Lagrange multipliers to find the point on the plane $x + 2y + z = 1$ that is closest to the origin. What is the distance of this point from the origin?

5. (10 marks) a) Sketch at least 3 level curves of the surface $f(x, y) = 9x^2 + 4y^2$.
b) Find an equation for the tangent plane to this surface at the point $(1/3, 1)$.
c) What is the directional derivative at $(1/3, 1)$ in the direction of the point $(-2/3, 3)$.

6. (15 marks) Use two methods to compute the flux of $\mathbf{F}(x, y, z) =< x, y, z >$ across the closed, positively oriented surface S where S is the paraboloid $x = y^2 + z^2$ which is capped by the disk $y^2 + z^2 \leq 4$, $x = 4$.

7. (10 marks) Compute the work done by the force $\mathbf{F}(x, y) =< y^2 \cos x, x^2 + 2y \sin x >$ where C is the triangle from $(0, 0)$ to $(2, 6)$ to $(2, 0)$ to $(0, 0)$. Hint: Use one of the theorems discussed in class.

Bonus: a) (4 marks) Given a vector field $\mathbf{F} =< P, Q, R >$ where P, Q and R are each functions of x, y and z, prove that div (curl $\mathbf{F}$) $= 0$.
b) (4 marks) If S is a closed surface, use the divergence theorem and the result from part a) to prove $\int\int_S \text{curl } \mathbf{F} \cdot d\mathbf{S} = 0$.