MAC 2313, Section 02 with Dr. Hurdal
Spring 2007 Final Exam Answers

1. $\frac{2}{9}(2^{3/2} - 1)$

2. π

3. Critical points/extrema occur at $(0,0)$, $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$, $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$, $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$, $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$.
The extrema are as follows:
maximum of $\frac{5}{2}$ at $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$, $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$; minimum of $-\frac{1}{2}$ at $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$, $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$.

4. $-\frac{8}{15}$

5. a) Level curves occur for $k \geq 1$ and are ellipses with minor axis along the x-axis and major axis along the y-axis.
b) $z = 4y - 3$
c) $\frac{16}{5}$
d) 4

6. $\frac{80}{3}$

7. a) $\mathbf{r}(t) =< 1 + 3t, -2t, 1 + t >$
b) Intersects at $(7, -4, 3)$ (when $t = 2$).
c) $\mathbf{r}(t) =< \frac{t^3}{2} + \frac{t^2}{2} + t + 3, -\frac{t^3}{3} + 2t + \frac{1}{3}, \frac{t^3}{6} + \frac{t^2}{2} - \frac{2}{3} >$

8. $-\frac{2}{3}\pi$

9. $\frac{32}{15}$

Bonus: The flow line is given by $\mathbf{r}(t) =< t - 2, \frac{t^2}{2} - 2t + 2 >$. This flow line corresponds to the parabola $y = \frac{x^2}{2}$ and so the sketch of the flow line begins at $(-2, 2)$, passes through the origin and continues in the direction of $(2, 2)$. Here is a description of what the vector field should look like (only a sketch is necessary): When $x < 0$, the vector field points to the right and downward (since the x-component of the vector field is always one and so in the direction of the positive x-axis; the y-component of the vector field is negative since x is negative and so points downward). When $x = 0$, the vector field points to the right with magnitude one (since the x-component of the vector field is always one and the y-component of the vector field is zero since $x = 0$). When $x > 0$, the vector field points to the right and upward (since the x-component of the vector field is always one and so in the direction of the positive x-axis; the y-component of the vector field is positive since x is positive and so points upward).